Separation of spatial and temporal components in rotating coil data analysis

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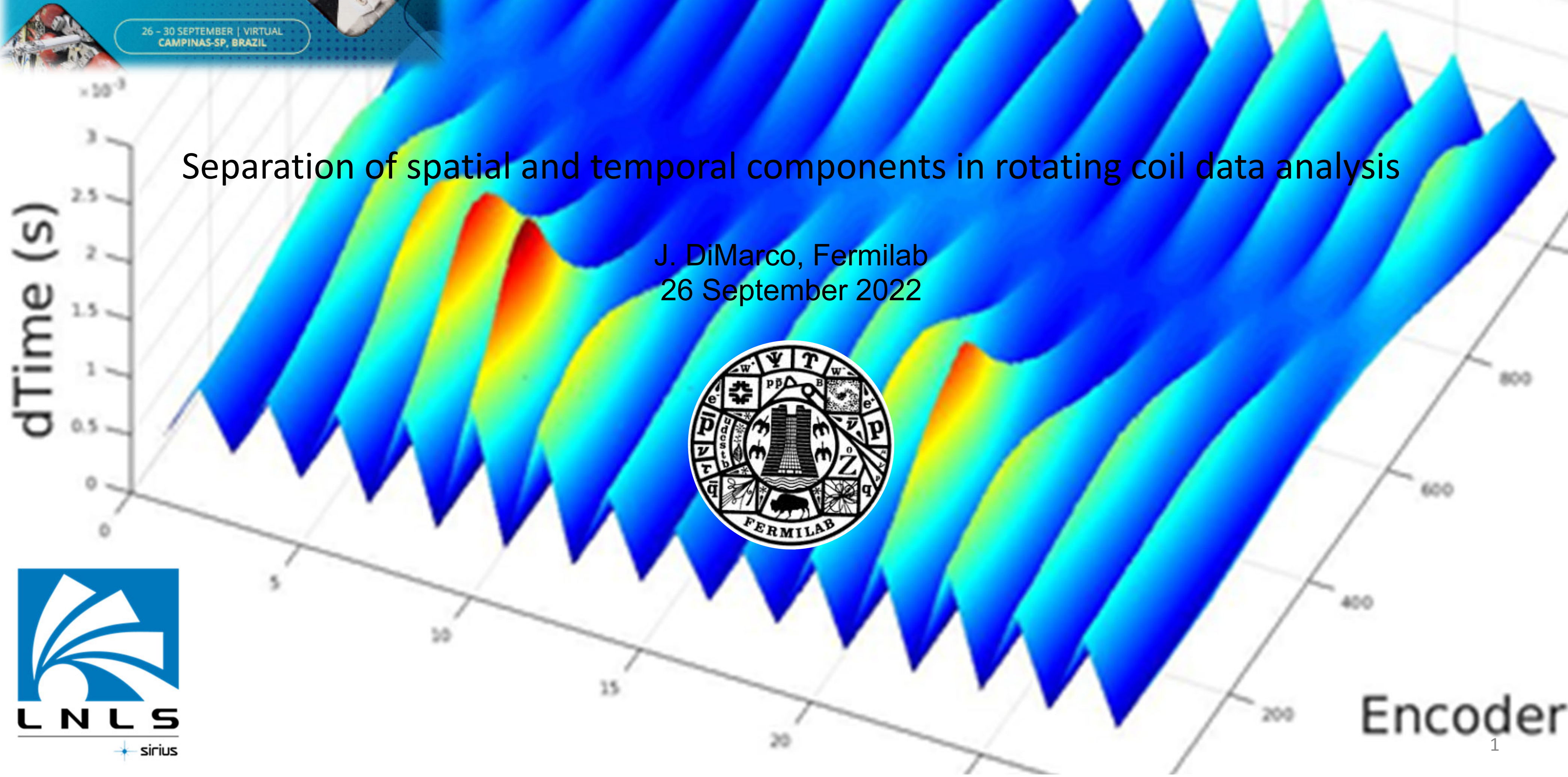

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Taking flux measurements with rotating coils is a standard technique for high accuracy measurements of field quality.

Harmonic analysis is applied, generally by taking data from rotating rectangular loops in fields of cylindrical symmetry and then applying Fourier transform.

Started to be used routinely in the 1950s- 1960s
(See e.g.

- W.C. Elmore, M.W. Garrett, "Measurement of two-dimensional fields, Part I: Theory", Rev. Sci. Instr., 25, (1954) 480-485.
- I.E. Dayton, F.C. Shoemaker, R.F. Mozley, "Measurement of two-dimensional fields, Part II: Study of a quadrupole magnet", Rev. Sci. Instr., 25, (1954) 485-489.
- C. Wyss, "A measuring system for magnets with cylindrical symmetry", Proc. 5th Int. Conf. on Magnet Technology (MT-5), Frascati, Italy (1975) 231-236.)

Analysis based on spatial dimensions

Flux expression in terms of multipole fields $(\boldsymbol{B_n}, \boldsymbol{A_n})$

$$\boldsymbol{\Phi(\theta)} = \mathbf{Re}\left[\sum_{n=1}^{\infty}(\boldsymbol{B_n} + \boldsymbol{iA_n}) * \boldsymbol{K_n} * \boldsymbol{e^{in\theta}}\right]$$

Probe geometry parameters

$$\boldsymbol{K_n} = \sum_{j=1}^{N_{wires}} \frac{\boldsymbol{L_j R}}{\boldsymbol{n}}\left(\frac{\boldsymbol{z_j}}{\boldsymbol{R}}\right)^{\boldsymbol{n}} * (-\mathbf{1})^{\boldsymbol{j}}$$

Complex Fourier Coefficients, F_n determined from flux measurements vs probe angle:

$$\Phi(\theta) = \sum_{n=1} F_n$$

Solve for field coefficients:

$$\boldsymbol{B_n} + \boldsymbol{iA_n} = \frac{\boldsymbol{F_n}}{\boldsymbol{K_n}}$$

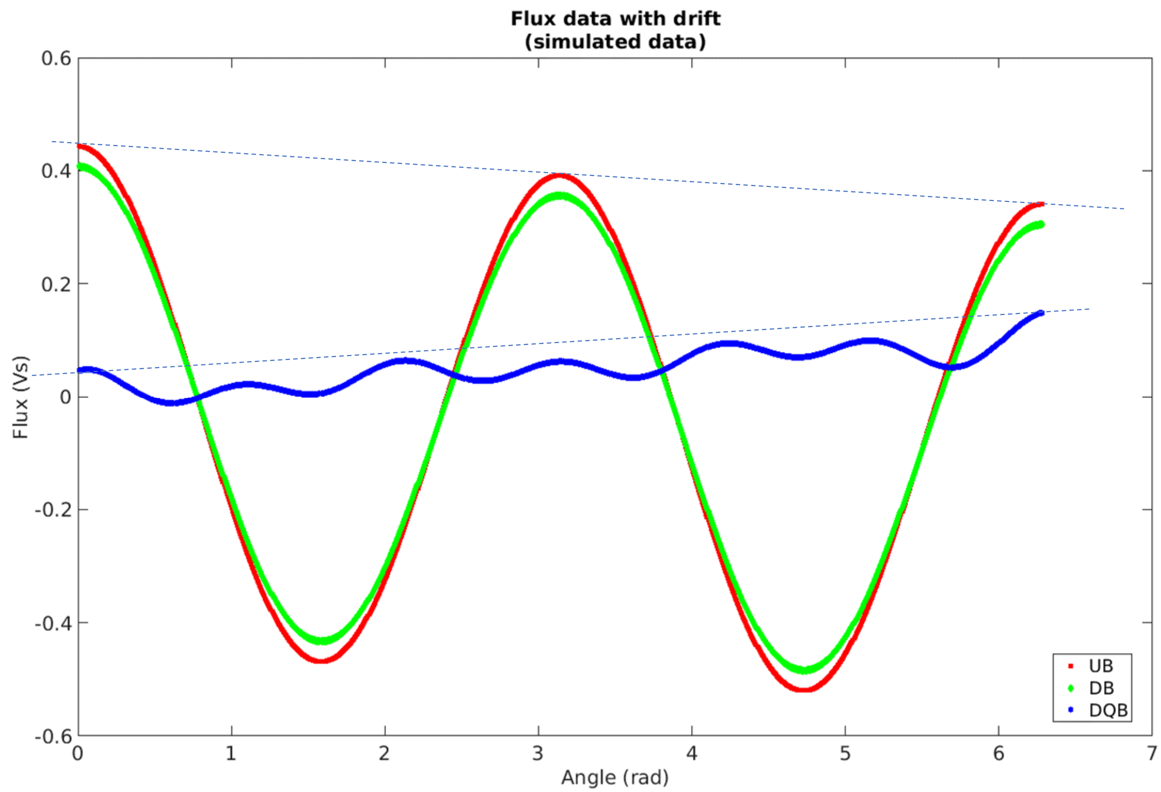
See e.g.
A. Jain, “Harmonic coils”, CAS Measurement and Alignment of Accelerator and Detector Magnets,Anacapri, Italy, 1997, CERN 98-05, p. 175.

Flux measurements may include time dependent effects. Before performing Fourier analysis, these effects, principally integrator drift, must be removed from signal

Typically drift slope found in one of several ways (but then always apply correction based on time stamp data for each sample)

- Single (first/last) point slope determination – perhaps use a fit to determine next point
- Overlap at least one point in next (consecutive) rotation to find slope
- Combine forward and reverse rotation data from probe

(Typically more complicated to do within single rotation)



There may also be other time-based effects present in the data – like particular electrical noise frequencies.

1) Matrix analysis can combine time analysis with spatial (angular) harmonic analysis to be able to separate these components in one operation

Rather than a FT function, solve a matrix that represents the flux data as a summation of sinusoids (as a FT does) as well as time dependent variables simultaneously.

Can restrict to harmonic orders of interest

For example to separate drift effects, first express the probe flux at each angle, θ, as a sum of the complex flux amplitudes φ_n,

$$\varphi(\theta) = Re\left[\sum_{n=1}^{15}(\varphi_n^{Re} + i\varphi_n^{Im}) * \mathrm{e}^{\mathrm{in\theta}}\right]$$

$$= \sum_{n=1}^{30} \varphi_n^{Re} * \cos(n\theta) - \varphi_n^{Im} * \sin(n\theta)$$

Combined matrix-based harmonic solution to the set of summation equations:

For the k angles of a rotation, we solve a $k \times 2n + 2$ matrix for orders of interest (e.g. k =1024, n = 30):

$n \rightarrow$ Solved for   Measured

$k \downarrow$

$$\begin{bmatrix} \cos(1\,\theta_1) & -\sin(1\,\theta_1) & \cdots & \cos(n\,\theta_1) & -\sin(n\,\theta_1) & t_1 & 1 \\ \cos(1\,\theta_2) & -\sin(1\,\theta_2) & \cdots & \cos(n\,\theta_2) & -\sin(n\,\theta_2) & t_2 & 1 \\ & & & \vdots & & & \\ & & & \vdots & & & \\ & & & \vdots & & & \\ \cos(1\,\theta_k) & -\sin(1\,\theta_k) & \cdots & \cos(n\,\theta_k) & -\sin(n\,\theta_k) & t_k & 1 \end{bmatrix} \begin{bmatrix} \varphi_1^{Re} \\ \varphi_1^{Im} \\ \vdots \\ \varphi_n^{Re} \\ \varphi_n^{Im} \\ S \\ O \end{bmatrix} = \begin{bmatrix} \Phi(\theta_1) \\ \Phi(\theta_2) \\ \vdots \\ \vdots \\ \vdots \\ \Phi(\theta_k) \end{bmatrix}$$

where the time of the flux sample, t_k (which may be substantially non-uniform), and unity columns allow simultaneous solution of the slope (S) and offset (O) together with the flux amplitudes of the signal.

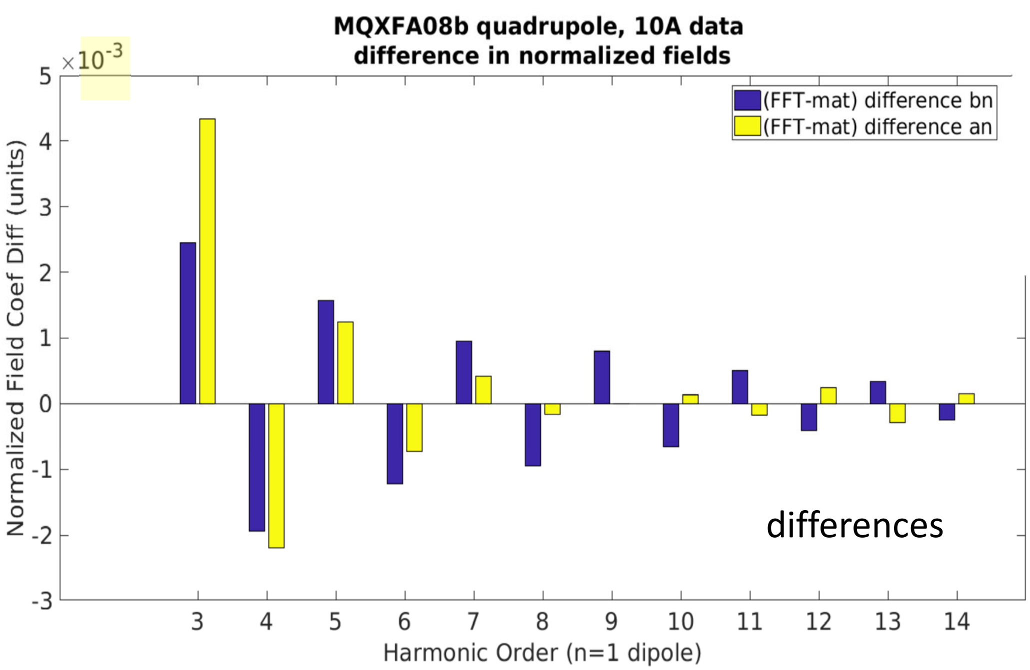
(should give 'best' removal of integrator drift based on a single rotation since incorporating both slope and actual harmonics)

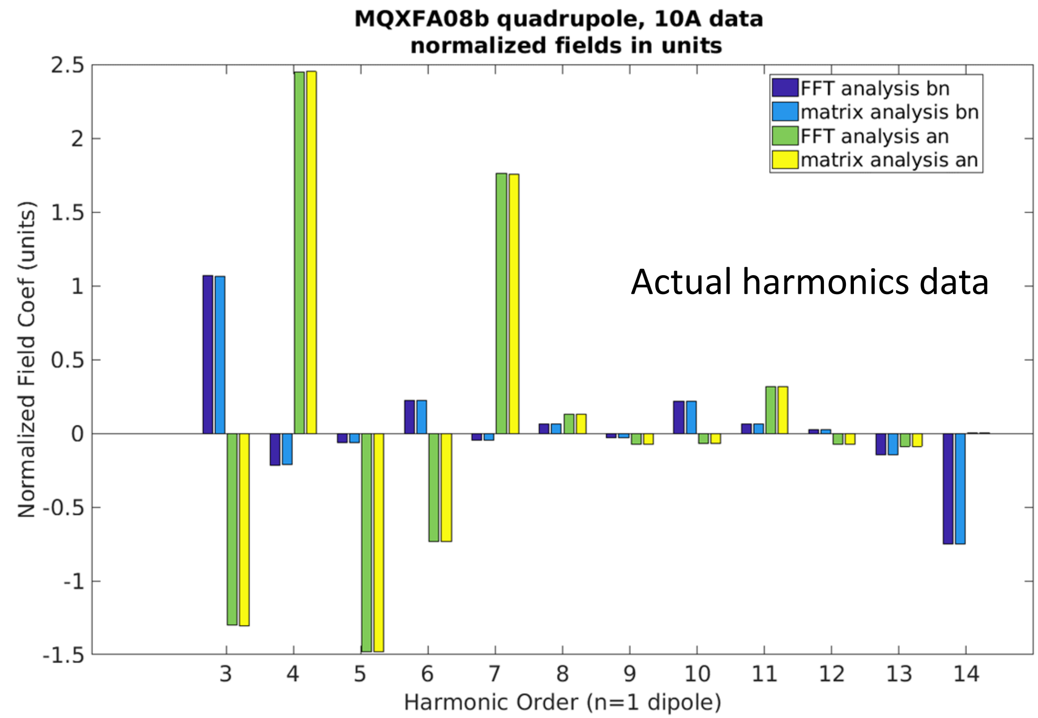
Note:

- Offset term is needed since otherwise any offset would try to be fit using cosine/sin terms.
- Same matrix is reused for each rotation (except time column)

Example comparing standard slope removal+FFT vs the matrix analysis in actual data*:

Differences are few mUnit





Techniques compare well with each other – at this level not clear which is better

(*MQXFA08b data courtesy Xiaorong Wang)

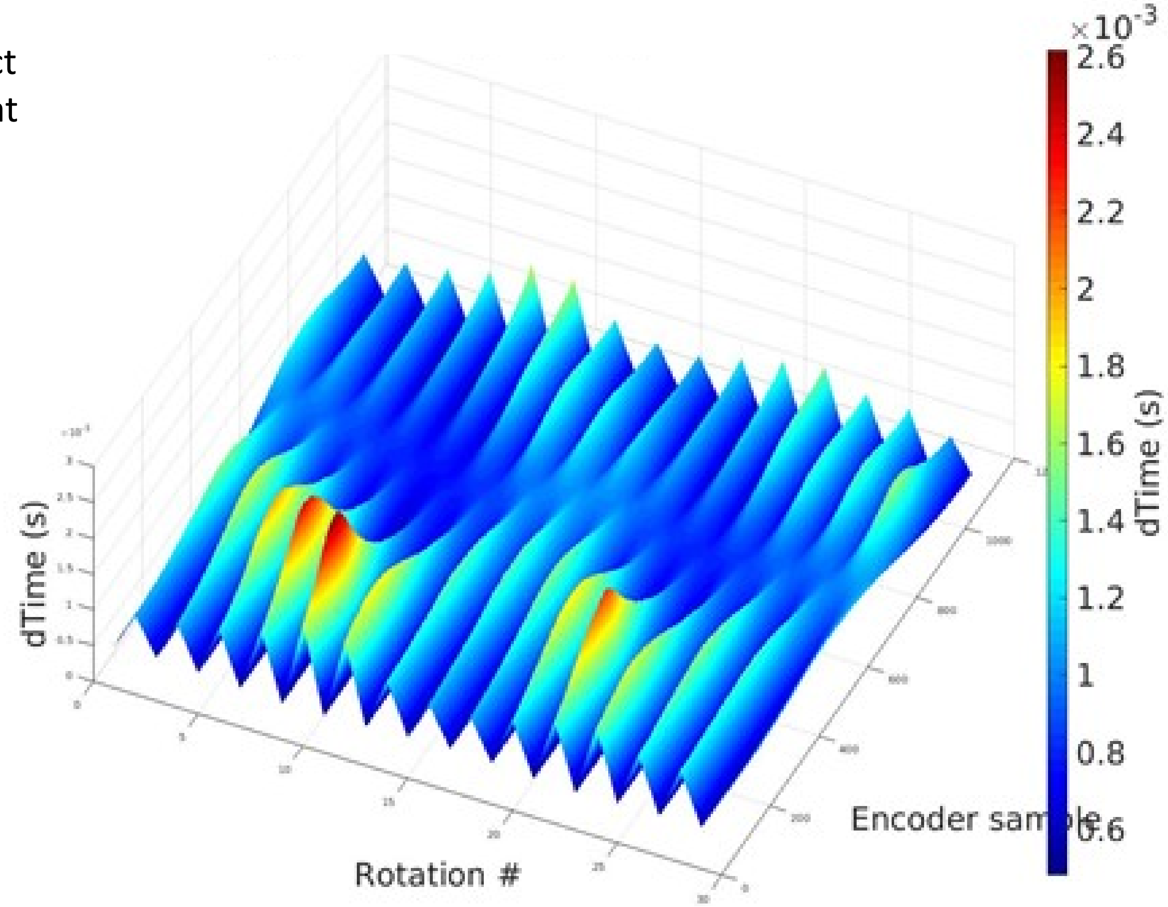
Difference was small for data on previous slide –
but will be measuring in a situation with large rotation variation

Measurements of HiLumi LHC Upgrade project quadrupole magnets for final testing at 1.9K at Fermilab will employ a 23m long, 6.35mm diameter, probe drive shaft rotating at 4Hz.

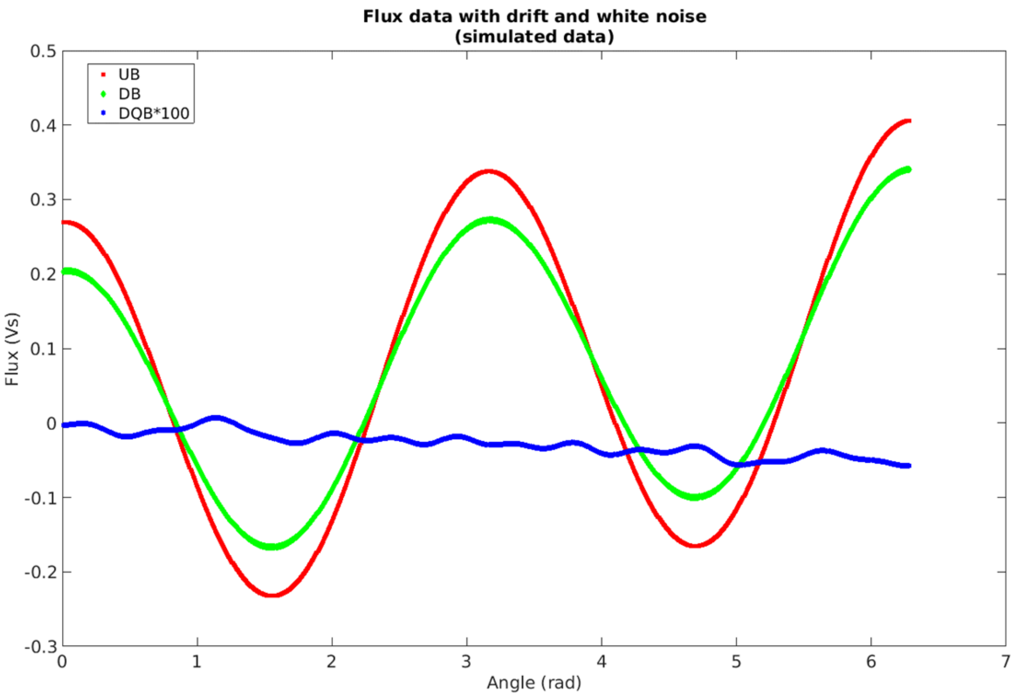
(more on this in a later talk)

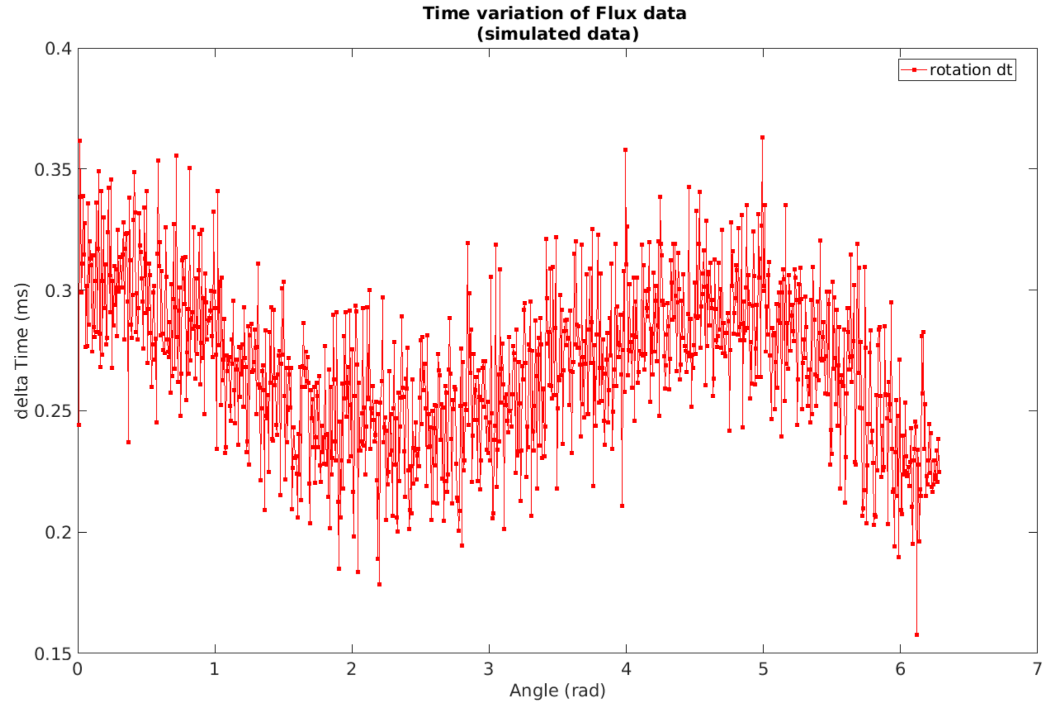
Have oscillations among sets of rotations, as well as large variations within a rotation.

➔ Try to simulate this data to see if this kind of variation causes any problems in analysis.



Simulate data at 4Hz
- Large time variation in rotation (~30%)
- Large flux drift slope (couples with time variation since drift is time-based)
- White noise added to signal (up to 1uVs)

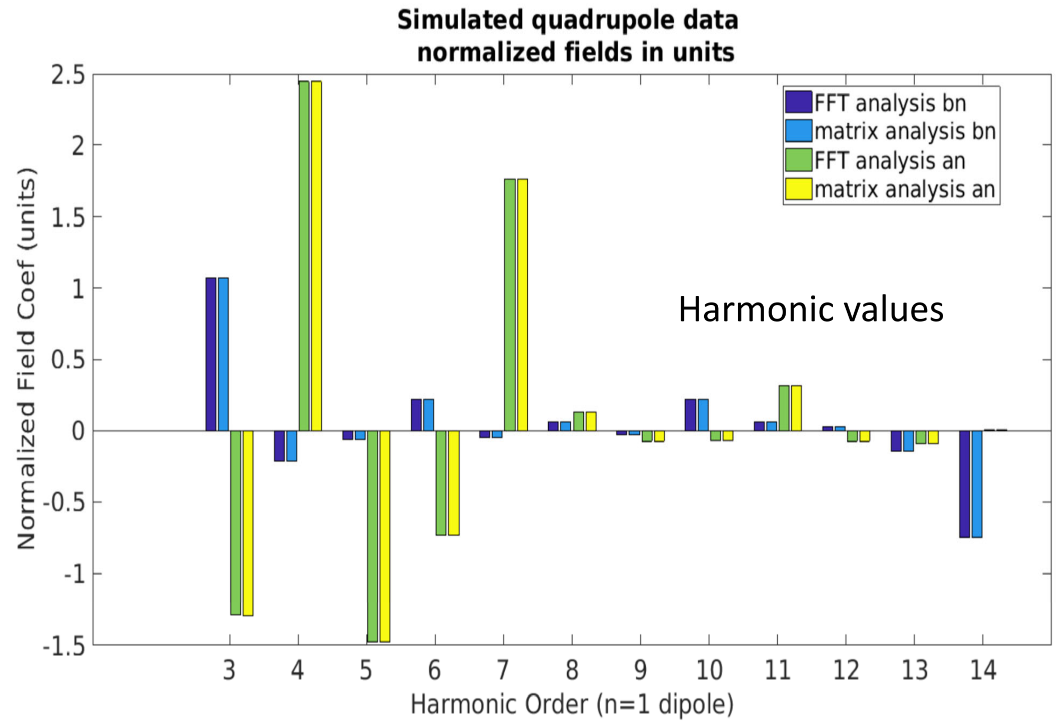




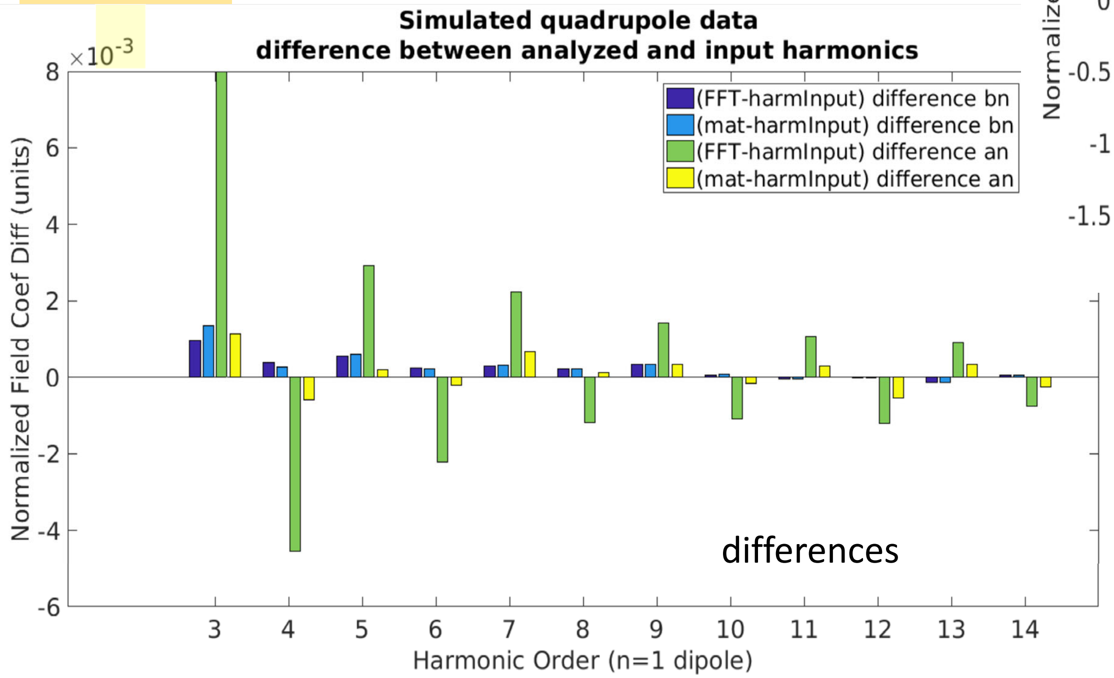
Errors are still small – can compare with simulation input

< 8 mUnits for drift/FFT analysis and less for mat-based technique

(note: the drift/FFT is also working on a single rotation)



< 10 mUnit



- Matrix analysis gives comparable results (e.g. depending on noise, one may be slightly better than another)
- Techniques compare well with each other and with known input

2) Matrix analysis can also separate unwanted noise frequencies from data:

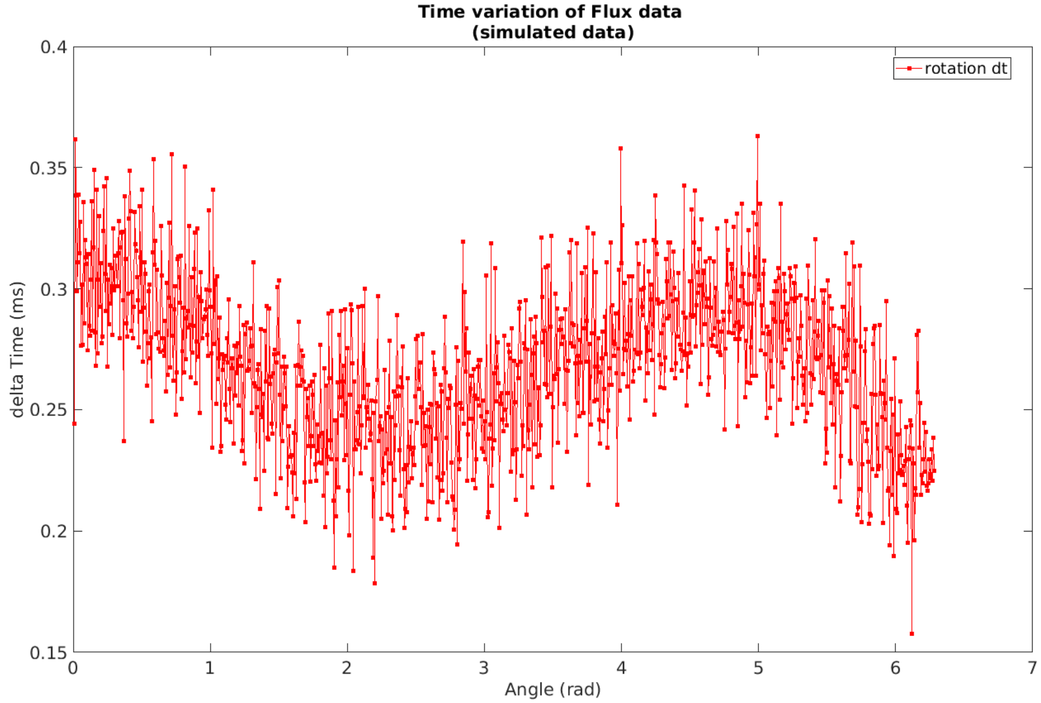
Columns with $cos(\omega t_k)$, $-\sin(\omega t_k)$ are inserted in the matrix to solve the amplitudes, N_ω^{Re}, N_ω^{Im}, associated with ω:

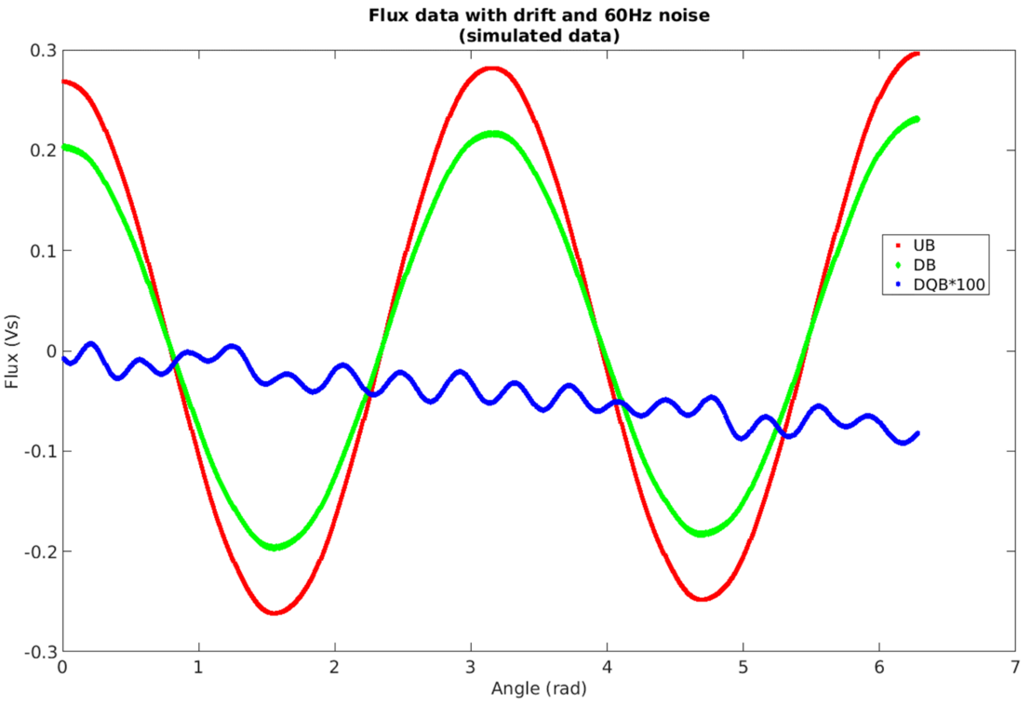
$$\begin{bmatrix} \cos(1\,\theta_1) & -\sin(1\,\theta_1) & \cdots & \cos(n\,\theta_1) & -\sin(n\,\theta_1) & \cos(\omega t_1) & -\sin(\omega t_1) & t_1 & 1 \\ \cos(1\,\theta_2) & -\sin(1\,\theta_2) & \cdots & \cos(n\,\theta_2) & -\sin(n\,\theta_2) & \cos(\omega t_2) & -\sin(\omega t_2) & t_2 & 1 \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ & & & & \vdots & & & & \\ \cos(1\,\theta_k) & -\sin(1\,\theta_k) & \cdots & \cos(n\,\theta_k) & -\sin(n\,\theta_k) & \cos(\omega t_k) & -\sin(\omega t_k) & t_k & 1 \end{bmatrix} \begin{bmatrix} \varphi_1^{Re} \\ \varphi_1^{Im} \\ \vdots \\ \varphi_n^{Re} \\ \varphi_n^{Im} \\ N_\omega^{Re} \\ N_\omega^{Im} \\ S \\ O \end{bmatrix} = \begin{bmatrix} \Phi(\theta_1) \\ \Phi(\theta_2) \\ \vdots \\ \vdots \\ \vdots \\ \Phi(\theta_k) \end{bmatrix}$$

This may be useful in our measurements - coil rotates at ~4 Hz, and n = 14 is of interest - the signal starts getting close to the 60 Hz AC line-cycle frequency.

Simulation #2 at 4Hz

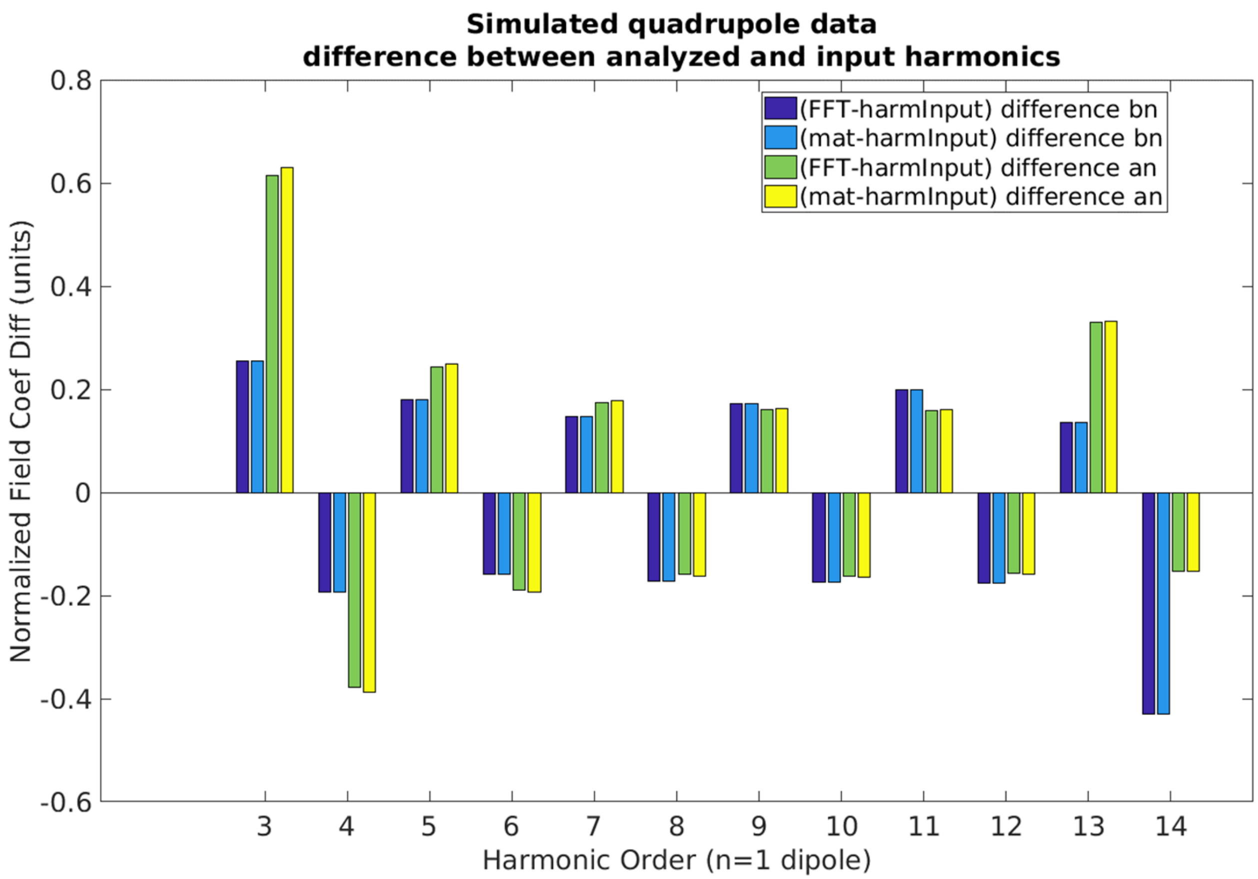
- Large time variation in rotation
- Large flux drift slope (couples with time variation since drift is time-based)
- Small amount of white noise in signal
- Large 60Hz noise presence





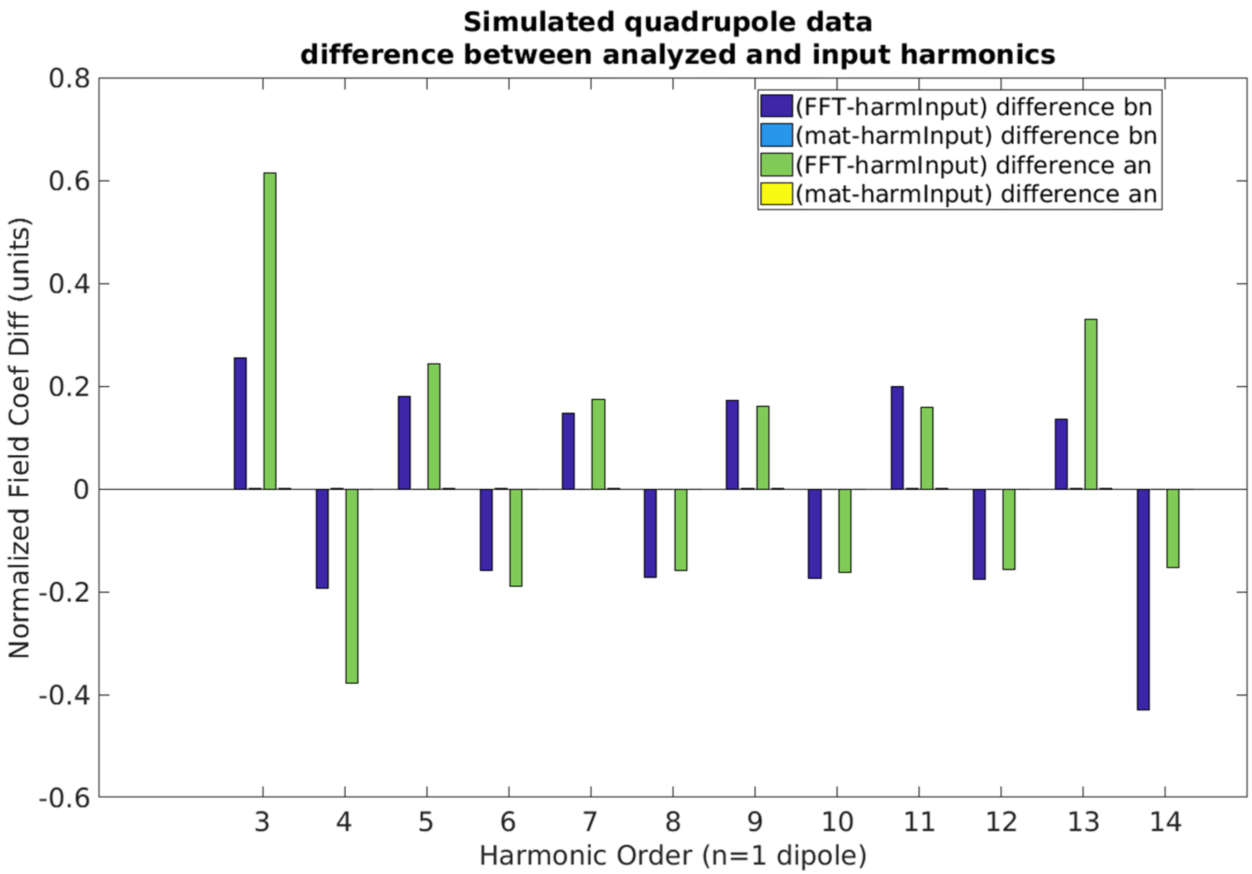
Errors are now large – **fractions of a unit** (~0.4 units for n=14) – not restricted to n=14

(the matrix result at this point does NOT include removal of the line frequency)



60Hz noise coupled with time variation can create large spurious harmonics
(which are not compensated by slope correction)

Using matrix with frequency removal columns – 60Hz effects are mitigated



Again at milli-unit level for matrix-based technique

Summary/notes:

- A matrix-based analysis gives a straightforward way to perform an analysis of a single rotation (e.g. that may help with analysis/software structuring), and can be easily implemented.
- Analysis incorporating time-based factors with the spatial analysis compares well to standard slope removal + FT analysis
- Technique can also be used to remove time-based effects of a known external frequency (e.g. line-cycle noise)
- Since the harmonics are limited to a smaller range of orders (typically n< 40), in general solving the matrix is about the same speed as an FFT, except when the number of angular samples (k) gets large (> ~2000).
- The redundancy (k equations for 2n+2 unknowns) has an effect of a least squares fit among the samples.
- Furthermore, if desired, the Kn can be included as part of the harmonics-solving matrix (see presentation in IMMW21 (Grenoble)), so that fields directly result from the solution.
- May be useful in improving analysis of other rotating coil measurements having time effects, such as dynamic field measurements.