System Decoupling and Control for High-Dimensional, High-Throughput Orbit Feedback

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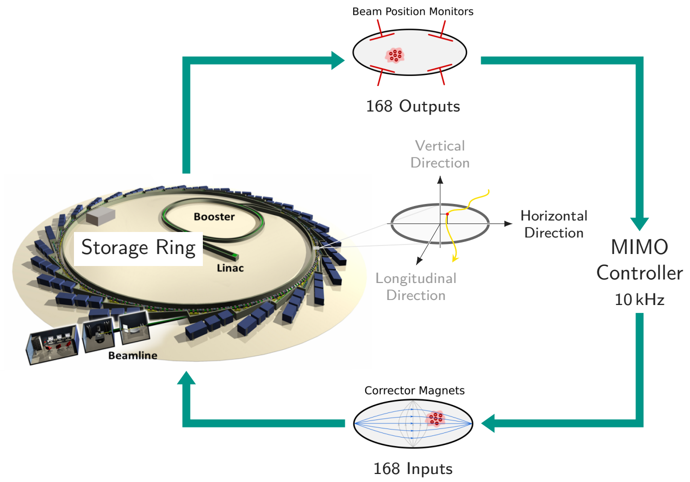
Diamond-I Plant & Goal



$$\boxed{y(s) = R\, g(s)\, u(s) + d(s)}$$

Laplace variable s: frequency–domain model. For sinusoidal analysis we set $s = j\omega$.

What each term means

- R: orbit response matrix (ORM).
- $g(s)$: actuator dynamics.
- $d(s)$: external disturbances acting on the orbit.

Why feedback is needed

- End stations require a *stable photon beam* at source points; stability translates to data quality and uptime.
- Disturbances: ground/girder vibrations, power–supply ripple, RF/cooling-induced lines, slow thermal drifts and insertion–device motion.

Goal (Diamond-I)

- Decouple R via SVD $\Rightarrow$ one SISO loop per mode.
- Choose simple internal model control (IMC) targets per mode for robust disturbance rejection.

SVD of R: What Changes in the System

Plant (original coordinates):

$$\boxed{y(s) = R\, g(s)\, u(s) + d(s)}$$

SVD of ORM: $R = U \Sigma V^T, \quad U^T U = I,\ V^T V = I.$
$\Sigma = \mathrm{diag}(\sigma_1, \ldots, \sigma_r)$ with $\sigma_1 \geq \cdots \geq \sigma_r > 0.$

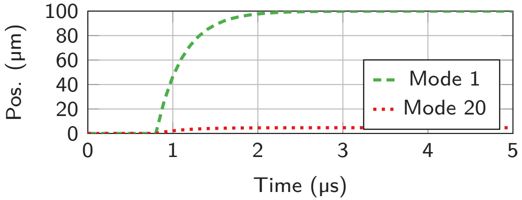
Modal transform: $\tilde{y} = U^T y,\ \tilde{u} = V^T u,\ \tilde{d} = U^T d.$

Apply the transform:

$$U^T y(s) \ = \ U^T R\, g(s)\, u(s) + U^T d(s) \ \Rightarrow \ \boxed{\tilde{y}(s) = \Sigma\, g(s)\, \tilde{u}(s) + \tilde{d}(s)}$$

Per mode i:

$$\tilde{y}_i(s) = \sigma_i\, g(s)\, \tilde{u}_i(s) + \tilde{d}_i(s).$$



Internal Model Control on SVD Modes

Per–mode plant (from SVD): $\tilde{y}_i(s) = \sigma_i\, g(s)\, \tilde{u}_i(s) + \tilde{d}_i(s)$.

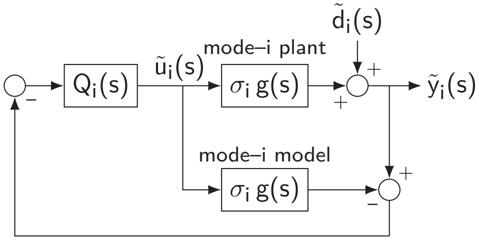
IMC filter (same form for every mode):

$$Q_i(s) \;=\; \big(\sigma_i\, g(s)\big)^{-1}\, T(s),$$

where $T(s)$ is a low–pass factor chosen for bandwidth and robustness.

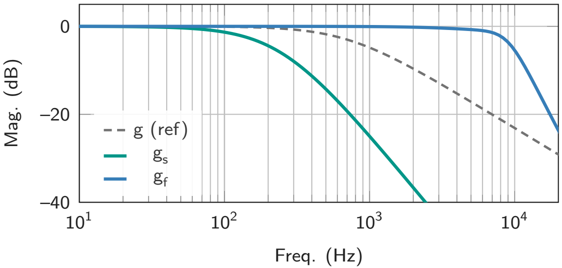
Closed–loop idea: with the nominal copy of $\sigma_i g(s)$ in the IMC path, the disturbance–to–output shape per mode is set mainly by $1 - T(s)$.

Back–map to actuators: $u = V\,\tilde{u}$ (right singular–vector basis).



Diamond-II Upgrade: Plant Form

Parameter	Diamond-I	Diamond-II
n_u (actuators)	168 (single array)	$396 = 252$ slow $+ 144$ fast
n_y (BPMs)	168	252
f_s	10 kHz	100 kHz
Closed-loop bandwidth	$\sim$ 300 Hz	slow+fast modes $\sim$ 1 kHz slow-only modes $\sim$ 100 Hz
Time delay	$\sim$ 700 μs	$\leq$ 100 μs



Corrector dynamics

$$y(s) = R_s\, g_s(s)\, u_s(s) \;+\; R_f\, g_f(s)\, u_f(s) \;+\; d(s)$$

- Two actuator arrays: **slow** $(R_s,\ g_s(s))$ with $n_s = 252$, **fast** $(R_f,\ g_f(s))$ with $n_f = 144$; outputs $n_y = 252$.
- **Design goal:** split control effort: slow array for $\sim$ 0–100 Hz, fast array for $\sim$ 100 Hz–1 kHz and avoid deadband.

GSVD of $[R_s\ R_f]$: What It Provides

GSVD (shared output basis):

$$R_s = X \begin{bmatrix} \Sigma_s & 0 \\ 0 & I \end{bmatrix} U_s^T, \qquad R_f = X \begin{bmatrix} \Sigma_f \\ 0 \end{bmatrix} U_f^T,$$

with $U_s^T U_s = I$, $U_f^T U_f = I$ and X invertible. Here $\Sigma_s = \mathrm{diag}(\sigma_{s,1}, \ldots, \sigma_{s,n_f}) \succ 0$, $\Sigma_f = \mathrm{diag}(\sigma_{f,1}, \ldots, \sigma_{f,n_f}) \succ 0$.

Complementarity (key property): for the first n_f directions,

$$\Sigma_s^2 + \Sigma_f^2 = I_{n_f} \quad \Longleftrightarrow \quad \sigma_{s,i}^2 + \sigma_{f,i} = 1, \quad i = 1, \ldots, n_f.$$

The remaining $n_y - n_f$ output directions correspond to the lower block: they appear through the identity block in R_s and have no contribution from R_f (slow-only in this basis).

Interpretation

- X^{-1} is a common output basis that simultaneously diagonalizes the two arrays.
- First n_f directions are slow+fast modes; the other $n_y - n_f$ directions are slow-only modes.
- For each slow+fast mode i, the pair $(\sigma_{s,i}, \sigma_{f,i})$ behaves as normalized mixing weights between slow and fast actuation.

Modal Coordinates and Modal Plant

Plant (original coordinates):

$$\boxed{y(s) = R_s\, g_s(s)\, u_s(s) \;+\; R_f\, g_f(s)\, u_f(s) \;+\; d(s)}$$

GSVD factors and modal coordinates:

$$R_s \;=\; X\begin{bmatrix}\Sigma_s & 0\\ 0 & I\end{bmatrix}U_s^T, \qquad R_f \;=\; X\begin{bmatrix}\Sigma_f\\ 0\end{bmatrix}U_f^T,$$

$$\tilde{y} \;=\; X^{-1}y, \qquad \tilde{d} \;=\; X^{-1}d, \qquad \tilde{u}_s \;=\; U_s^T u_s, \qquad \tilde{u}_f \;=\; U_f^T u_f.$$

Apply the transform to the boxed plant:

$$X^{-1}y(s) = X^{-1}R_s g_s(s) u_s(s) + X^{-1}R_f g_f(s) u_f(s) + X^{-1}d(s)$$

$$\Rightarrow \quad \boxed{\tilde{y}(s) = \begin{bmatrix}\Sigma_s & 0\\ 0 & I\end{bmatrix} g_s(s)\, \tilde{u}_s(s) \;+\; \begin{bmatrix}\Sigma_f\\ 0\end{bmatrix} g_f(s)\, \tilde{u}_f(s) \;+\; \tilde{d}(s)}$$

Per-mode view:

$$\tilde{y}_i(s) = \sigma_{s,i}\, g_s(s)\, \tilde{u}_{s,i}(s) + \sigma_{f,i}\, g_f(s)\, \tilde{u}_{f,i}(s) + \tilde{d}_i(s), \quad i = 1, \ldots, n_f,$$

$$\tilde{y}_j(s) = g_s(s)\, \tilde{u}_{s,j}(s) + \tilde{d}_j(s), \quad j = n_f + 1, \ldots, n_y.$$

IMC on GSVD Modes: Per-Mode Mid-Ranging

Goal per mode

- TISO modes ($i \le n_f$): target $T_f(s)$ with bandwidth ≈ 1 kHz.
- SISO modes ($j > n_f$): target $T_s(s)$ with bandwidth ≈ 100 Hz.

Per-mode controller (IMC form)

- *TISO* ($i \le n_f$): split the target

$$q_{s,i}(s) = \left(\sigma_{s,i}\, g_s(s)\right)^{-1} T_s(s), \qquad q_{f,i}(s) = \left(\sigma_{f,i}\, g_f(s)\right)^{-1} \left(T_f(s) - T_s(s)\right).$$

- *SISO* ($j > n_f$): slow only

$$q_j(s) = g_s(s)^{-1}\, T_s(s).$$

What this achieves (mid-ranging)

- Slow path tracks $T_s(s)$ for $\approx$ 0–100 Hz.
- Fast path adds exactly the $\left(T_f(s) - T_s(s)\right)$ remainder for $\approx$ 100 Hz–1 kHz.
- No deadband; each mode meets its (common) fast target with a clean slow/fast split.

Back to actuators

$$u_s = U_s\, \tilde{u}_s, \qquad u_f = U_f\, \tilde{u}_f.$$

Toward Multiple Arrays

Plant (3 slow arrays + fast):

$$y(s) \;=\; \sum_{i=1}^{3} R_{si}\, g_{si}(s)\, u_{si}(s) \;+\; R_f\, g_f(s)\, u_f(s) \;+\; d(s).$$

Adopted route:

- **Balancing input filters** to reduce to a 2-array form: $u_{\bar{s}i} = \dfrac{g_{si}(s)}{g_{\bar{s}}(s)}\, u_{si}(s) \Rightarrow y = R_s\, g_{\bar{s}}\, u_{\bar{s}} + R_f\, g_f\, u_f + d.$

If you are interested, consider attending this related session:
Fast orbit feedback using the GSVD for systems with multiple slow corrector arrays (THCG001), Sep 25, 2025, 2:00 PM.

Key Takeaways

- **Diamond-I:** Do an SVD of the orbit response matrix to get independent modes; design a simple per-mode IMC target and you're done.
- **Diamond-II (two arrays):** Use a GSVD of $[R_s \; R_f]$ to get a common output basis; each mode becomes either two-input (fast+slow) or slow-only, and IMC targets are set separately for the two groups.
- **Three slow arrays (different dynamics):** Equalise with simple input "balancing" filters to reduce back to the two-array case; apply the same GSVD+IMC recipe and back-map to physical actuators.

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Thank you!