H2, H∞, and beyond: for LIGO

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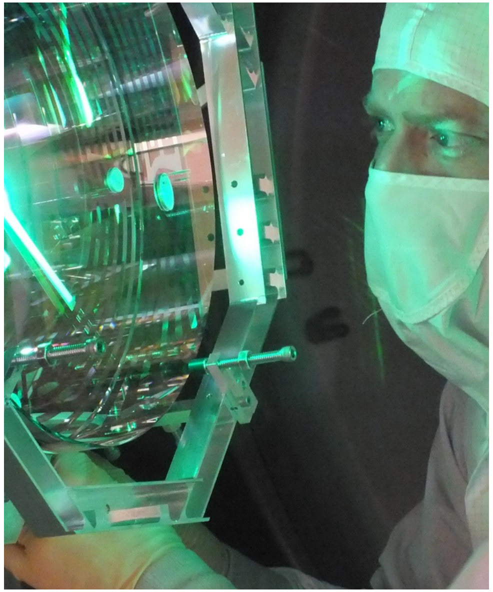
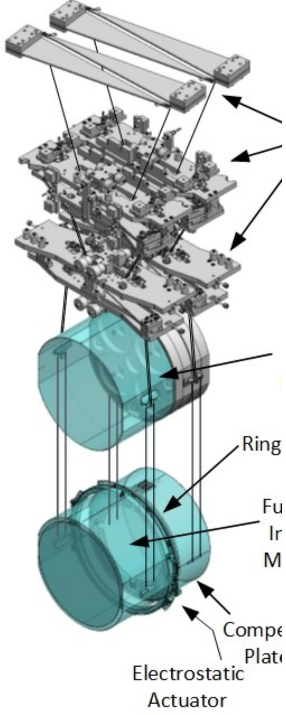
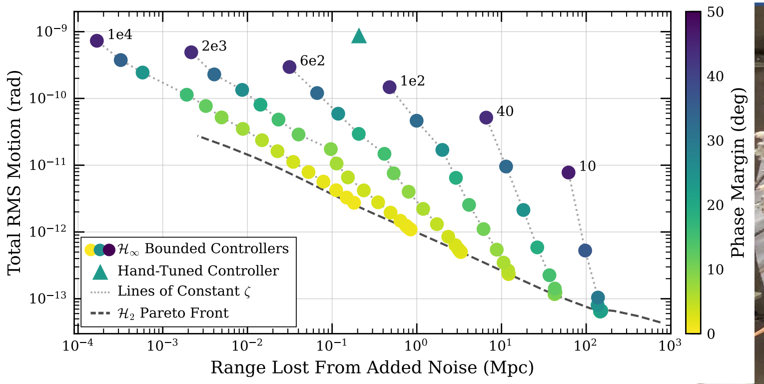
Caltech

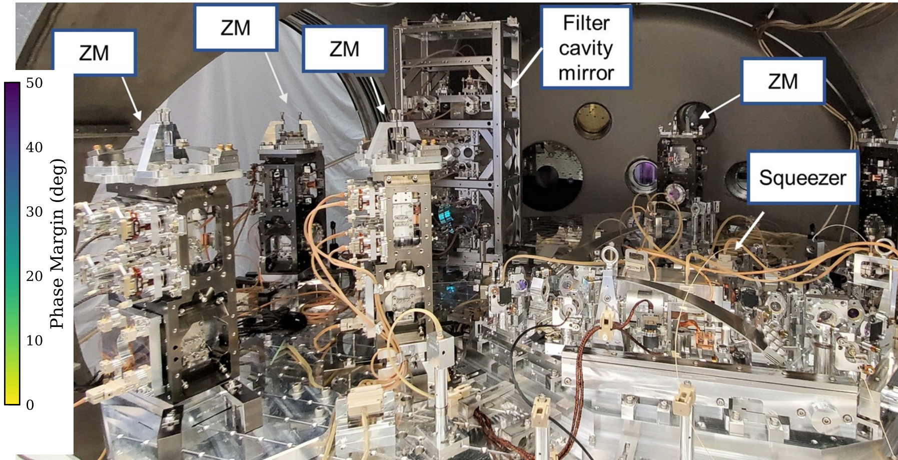
ICALEPCS
21 Sept 2025





How strongly can we probe the LIGO mirrors?





Is SISO Bode-criterion Control the best we can do?

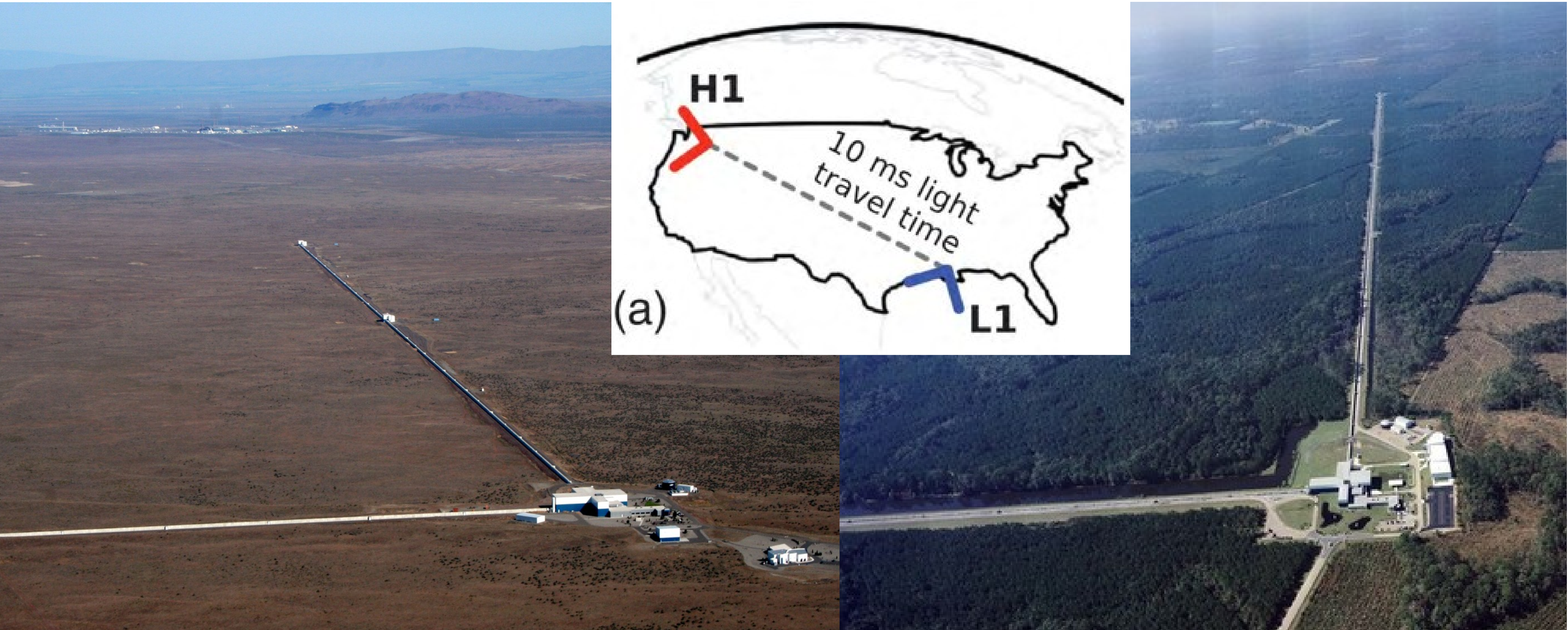
Roadmap

- Intro to LIGO, gravitational waves signals and control
- Case study, angular control noise injection.
  - Noise Shape and old design method
- Modernizing to optimal control
- Noise-optimal (L2 / LQG) with guaranteed phase margin ($H\infty$ bound)
- Views of the Pareto front

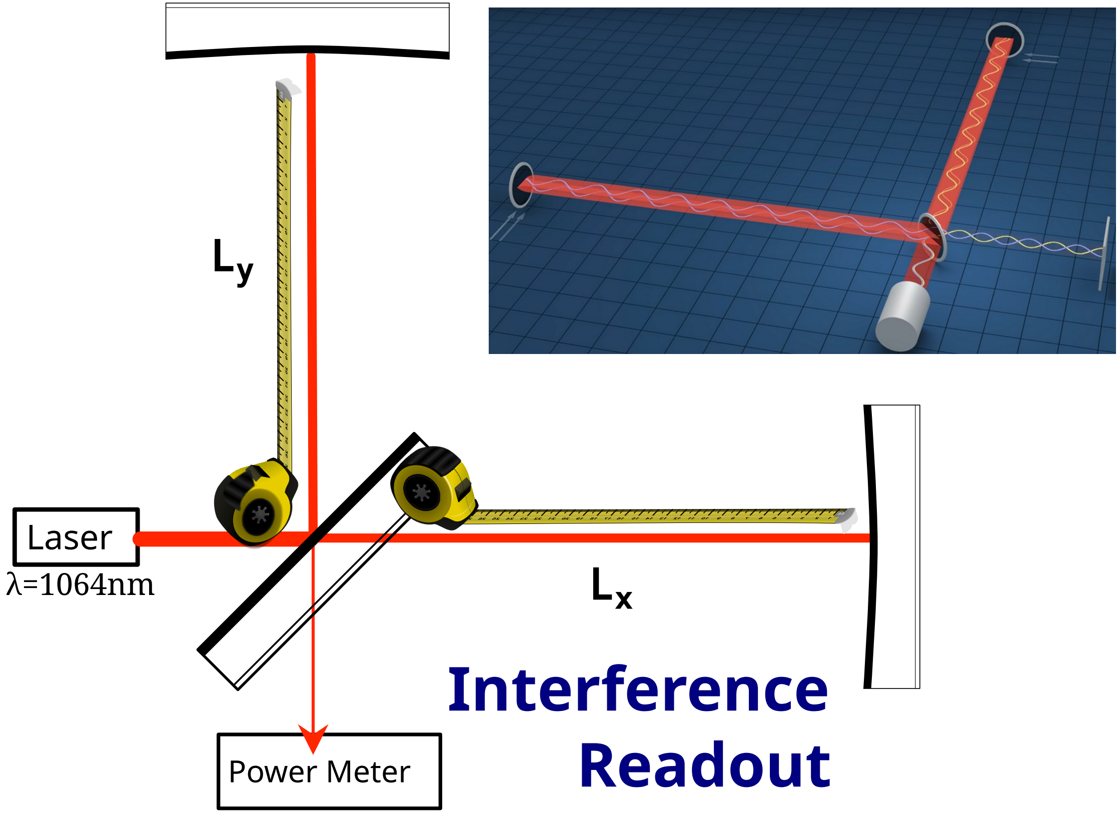
Two LIGO Observatories

each one a

laser interferometer with 4 km arms

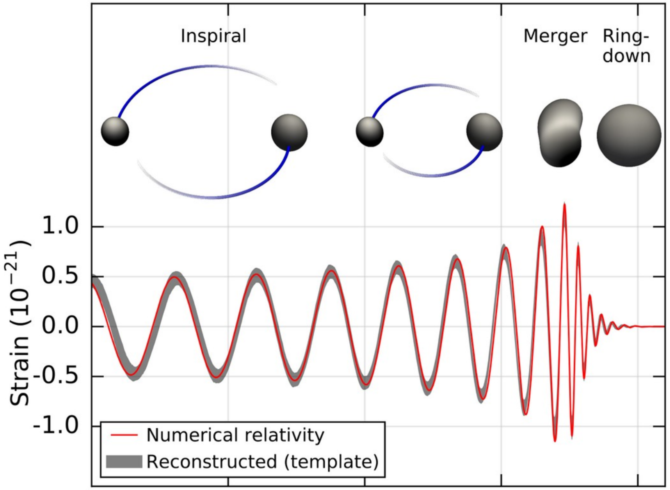


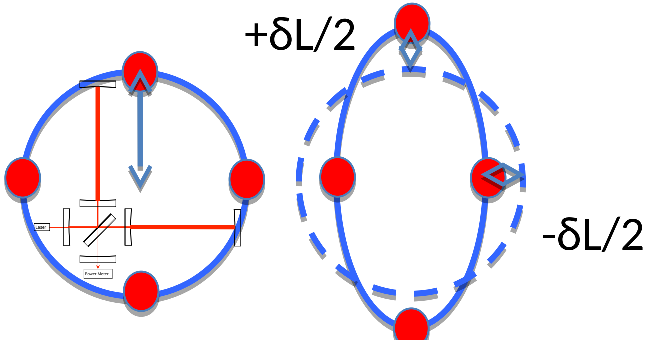
Gravitational Strain Transduction



Interference Readout

Phys. Rev. Lett. 116, 061102



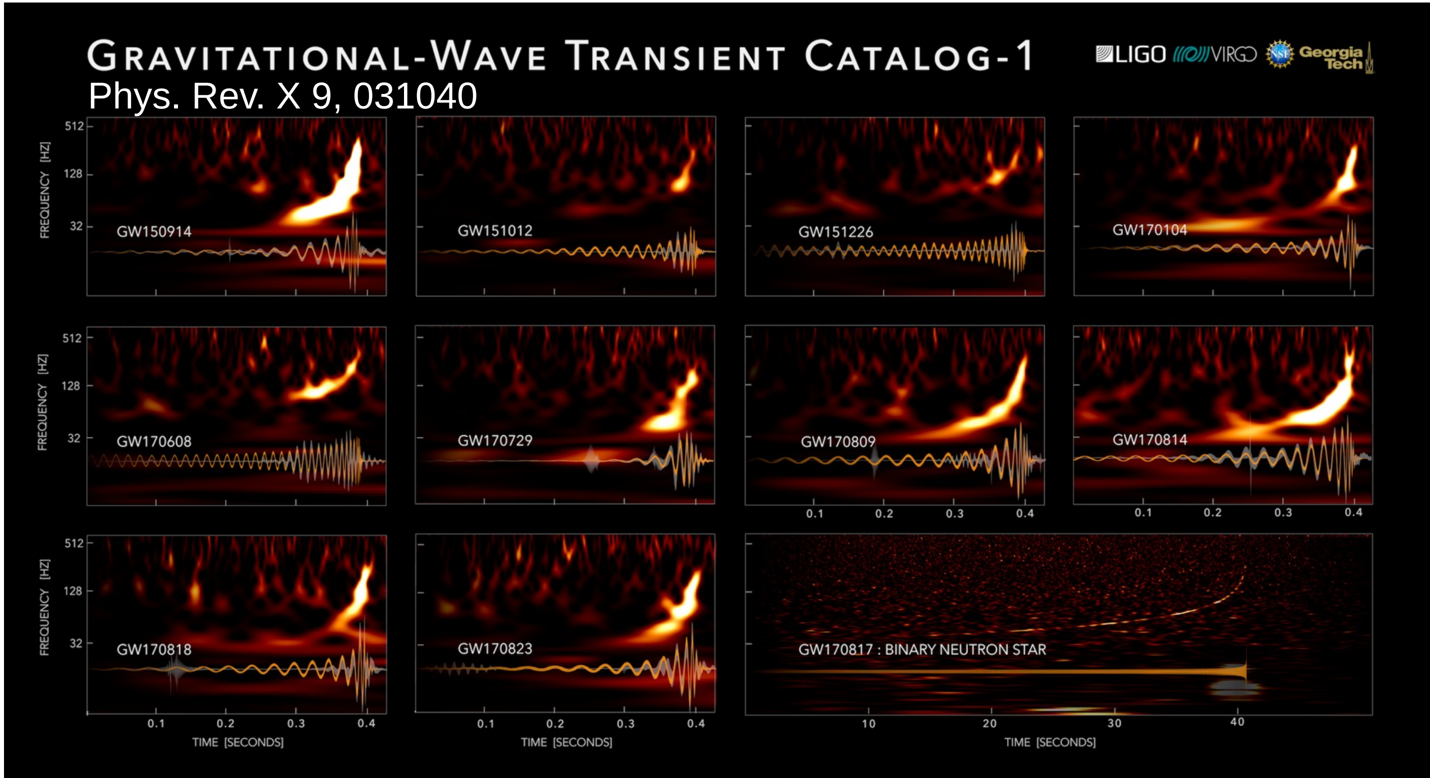


Science Signals

The population is made of individuals. Each is exciting!

Estimation of many-parameters → rich space for (quantum) optimal inference.

Control and hardware design should optimize for our astronomical inference!



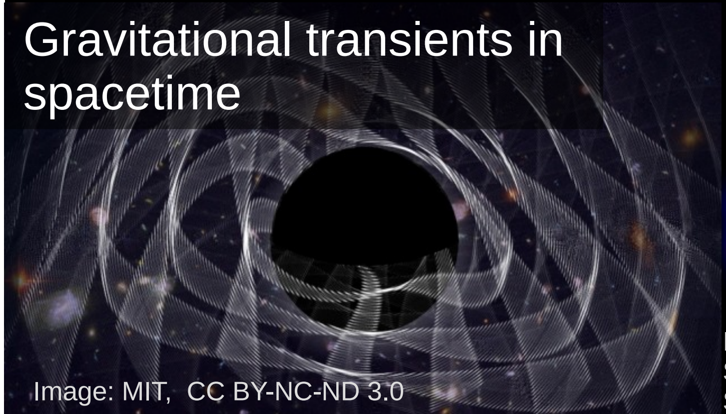


Image: MIT, CC BY-NC-ND 3.0

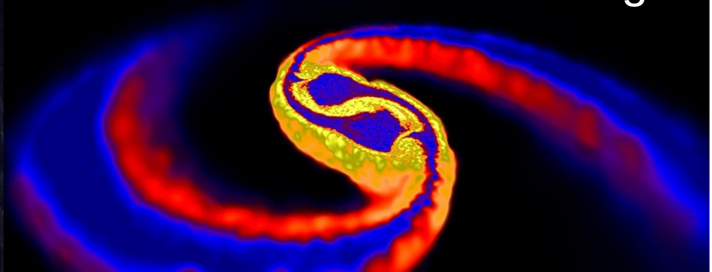
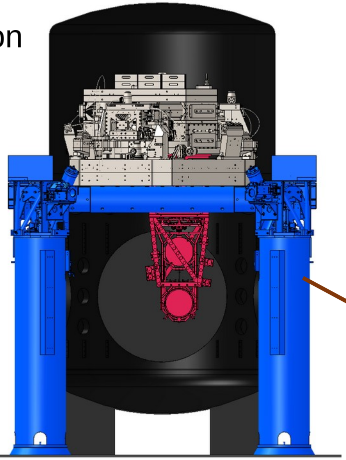


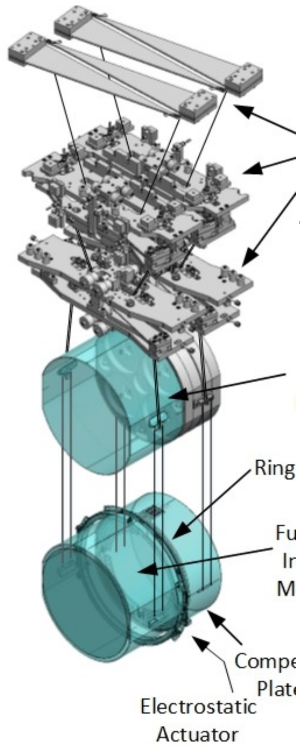
Image: Daniel Price (University of Exeter) and Stephan Rosswog (International University Bremen), Science 2006

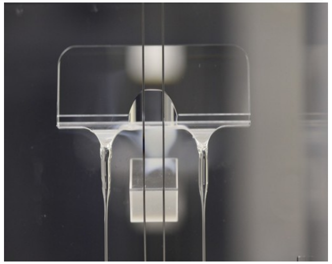
advanced LIGO’s noise sources

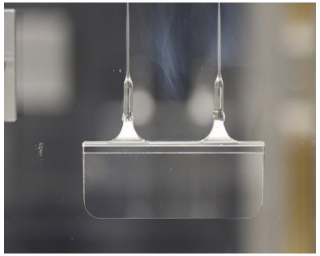
Active inertial isolation



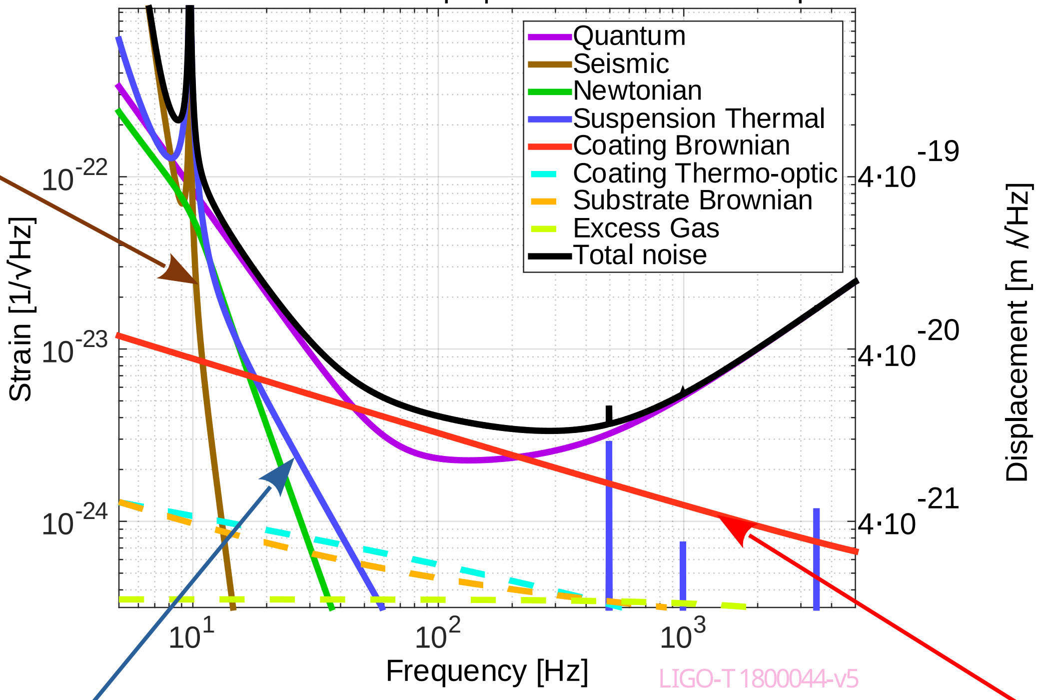
Passive quadruple Suspension





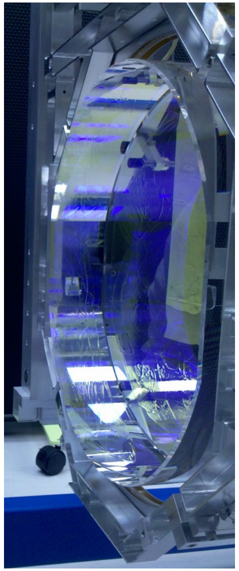


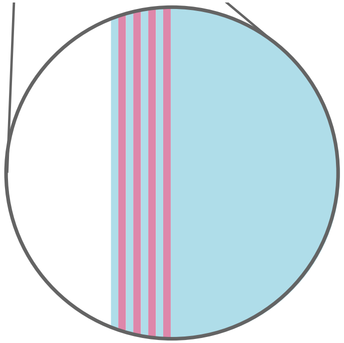
aLIGO Design Curve
1.4:1.4M. NSNS-173 Mpc | 30:30M. BHBH-1600Mpc



Fused Silica Suspension
Q~1 billion, 2 week ringdown

2nm smooth core optics,
~40ppm loss on 6cm beam





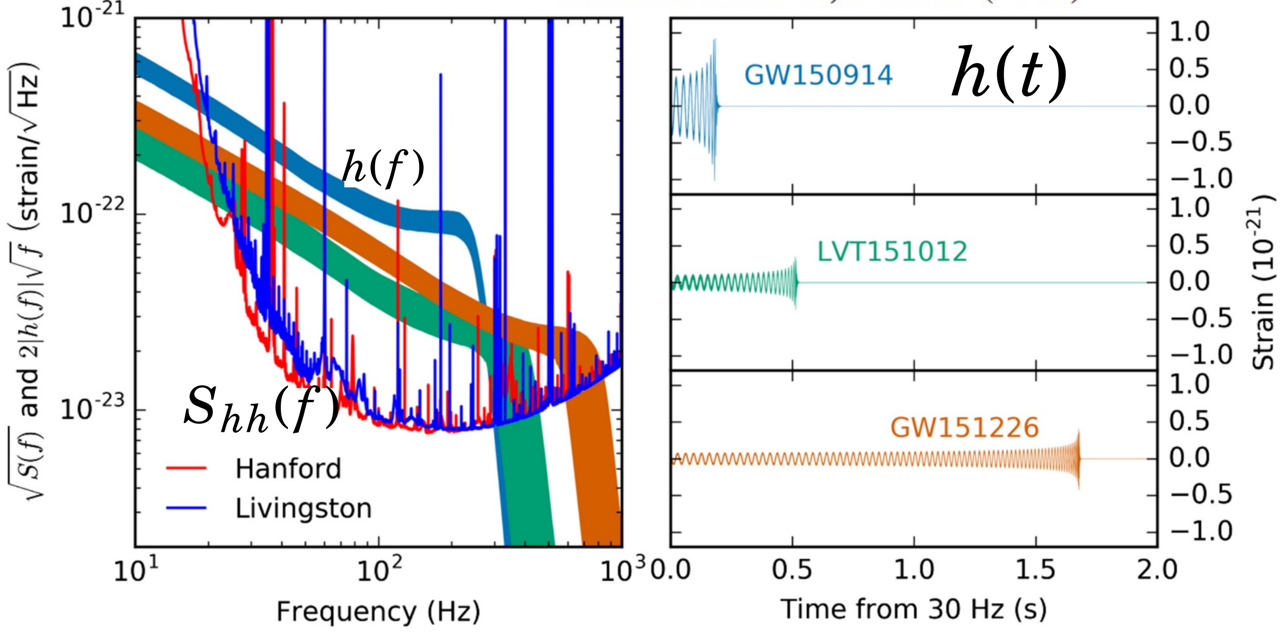
Coating materials minimize
Structural loss contributions
To thermal noises

Frequency Domain Signals

- Key Figure of merit is Signal-to-Noise
- Observing range proportional to SNR
  - 100’s millions of light years (big volume!)

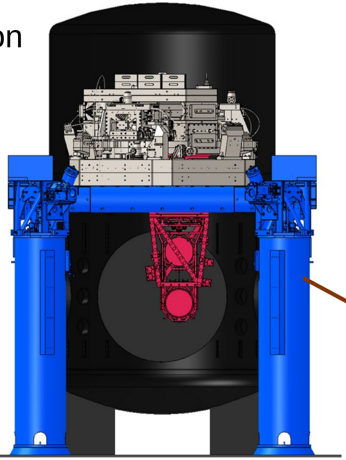
$$\mathrm{SNR}^2 = 4\int_0^\infty \frac{h^2(f)}{S_{hh}(f)}\mathrm{d}f$$

PHYS. REV. X **6,** 041015 (2016)



advanced LIGO’s noise sources

Active inertial isolation



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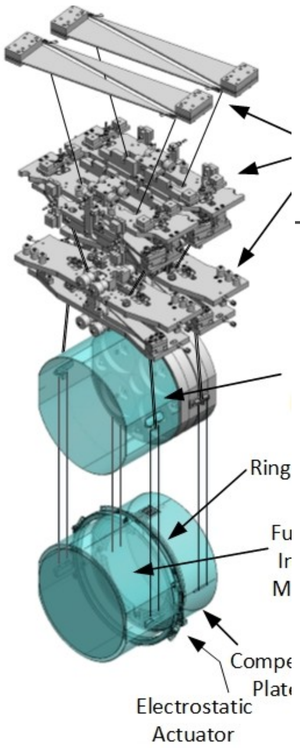

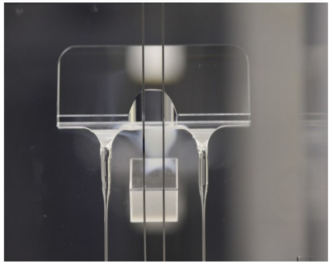

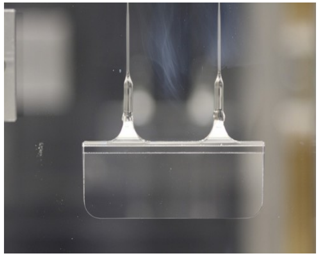

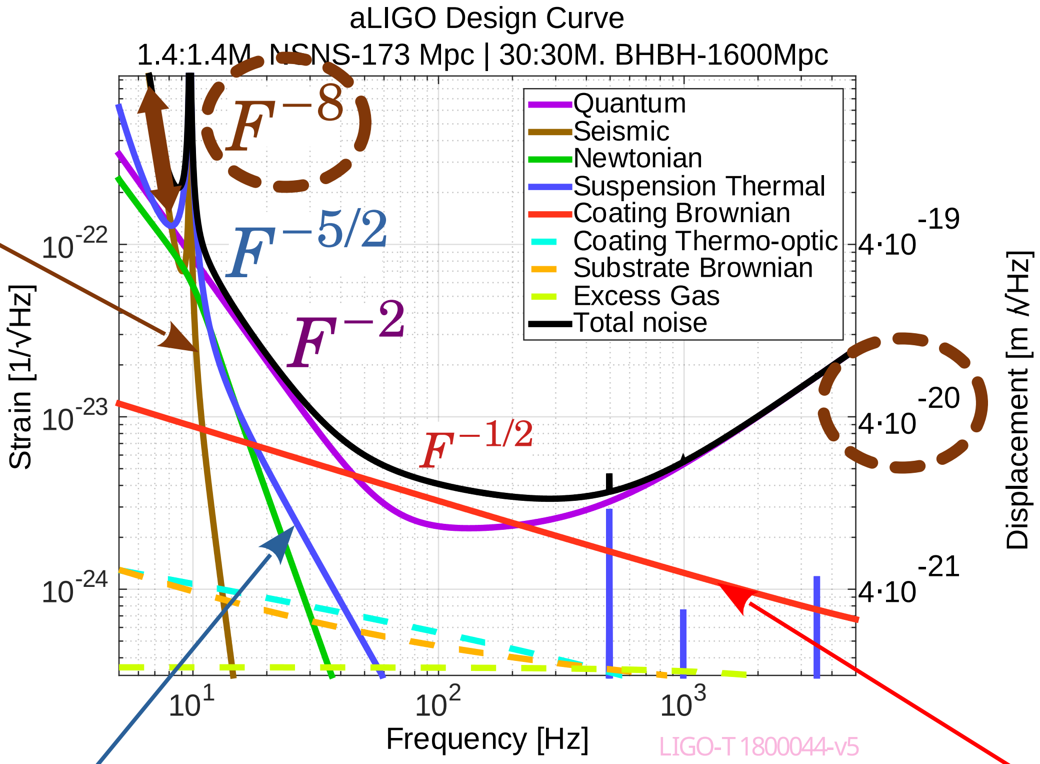

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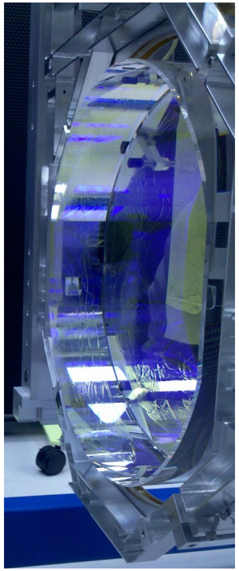
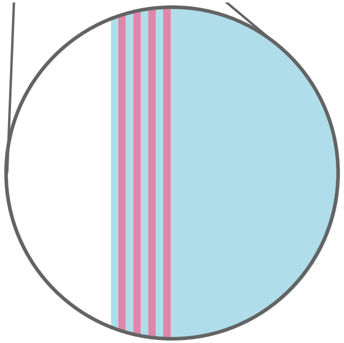
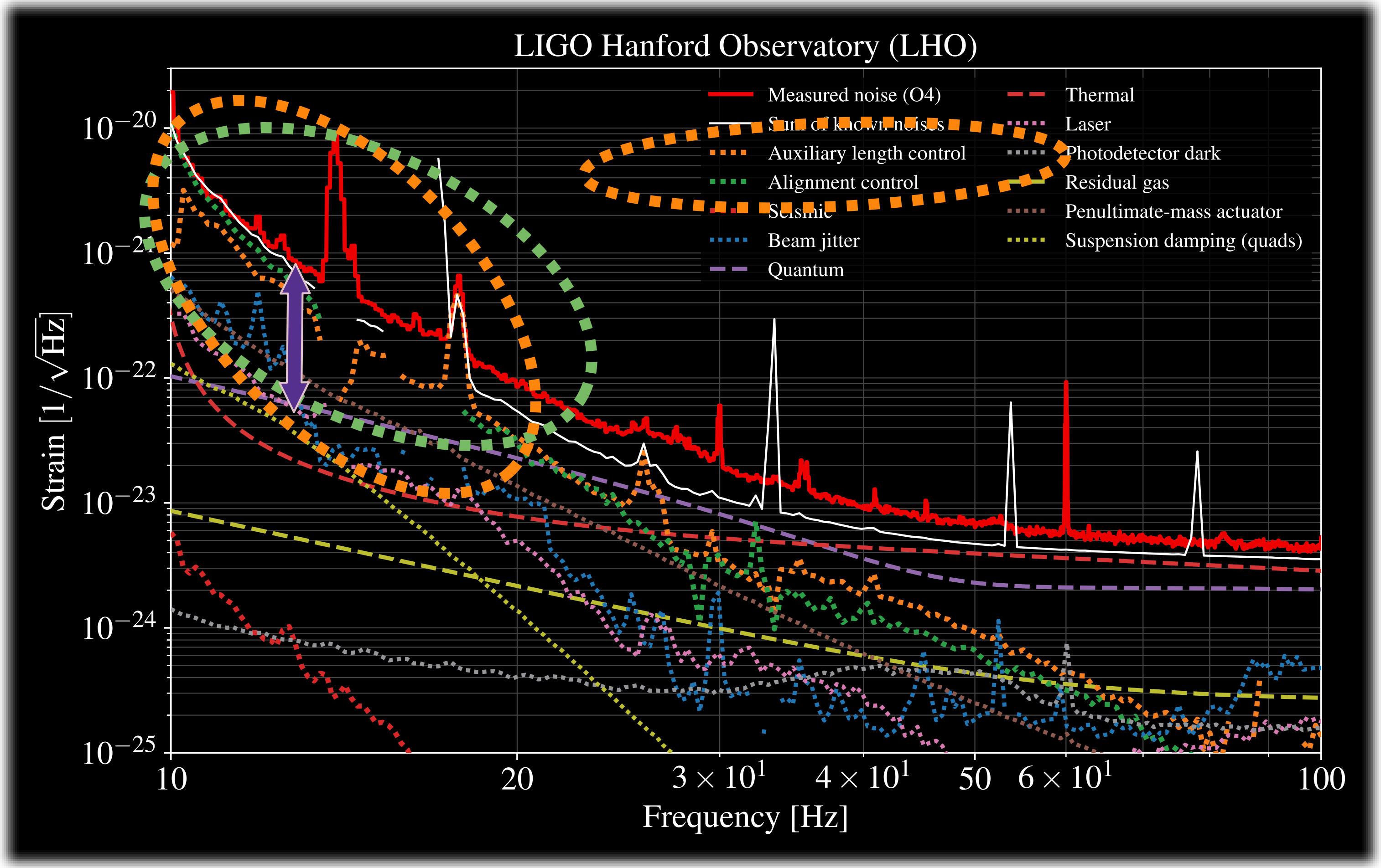




Coating materials minimize
Structural loss contributions
To thermal noises

Alignment Controls Noise

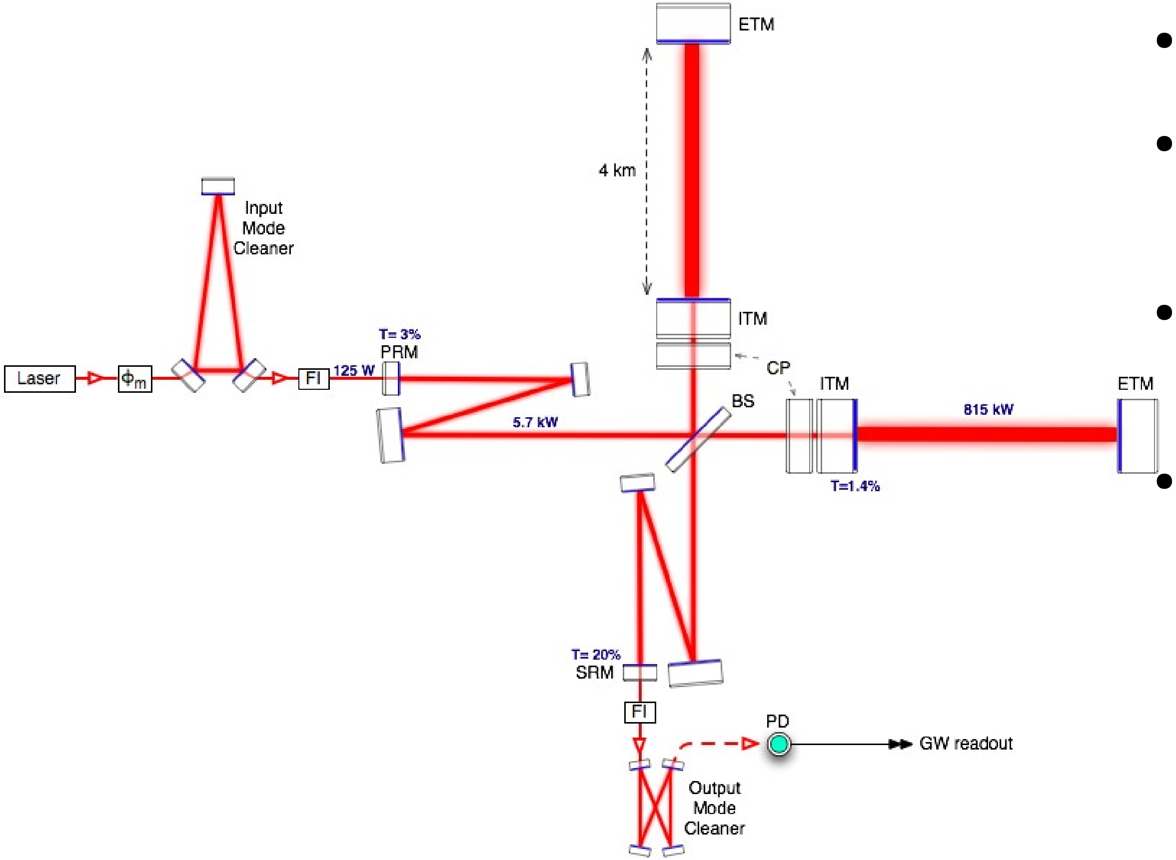
- Low frequencies are limited by noise from auxiliary degrees of freedom
  - Length
  - Alignment
- Loop shaping helps



Advanced LIGO detector performance in the fourth observing run

E. Capote[1,2,3,*,†], W. Jia[4,*], N. Aritomi[2,*], M. Nakano[3,5,*], V. Xu[4,6,*], R. Abbott[3], I. Abouelfettouh[2], R.X. Adhikari[3], A. Ananyeva[3] *et al.*

LIGO’s Optical Layout



- Many cavities/resonators
- Each mirror is 3 degrees of freedom in interferometer
- Each mirror has 18-24 degrees of freedom in suspensions
- Layers of control between seismic->suspension->optical

LIGO's Control Strategy

Diagonalize plant from models

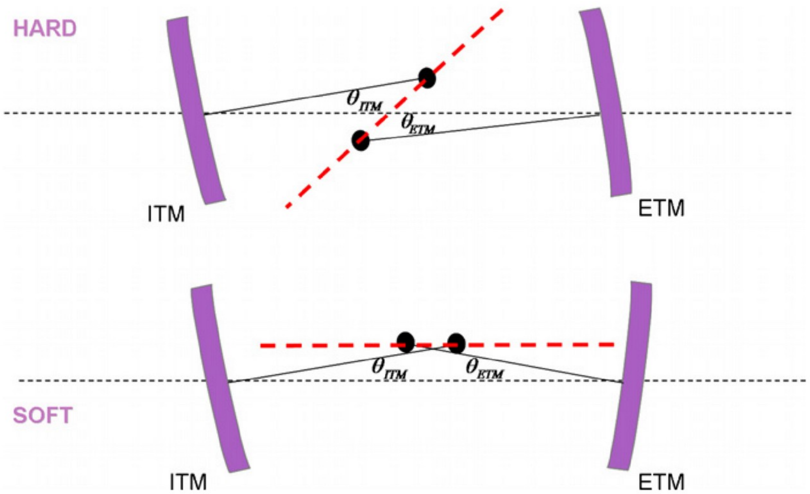
Design **SISO loops** using Bode Plots

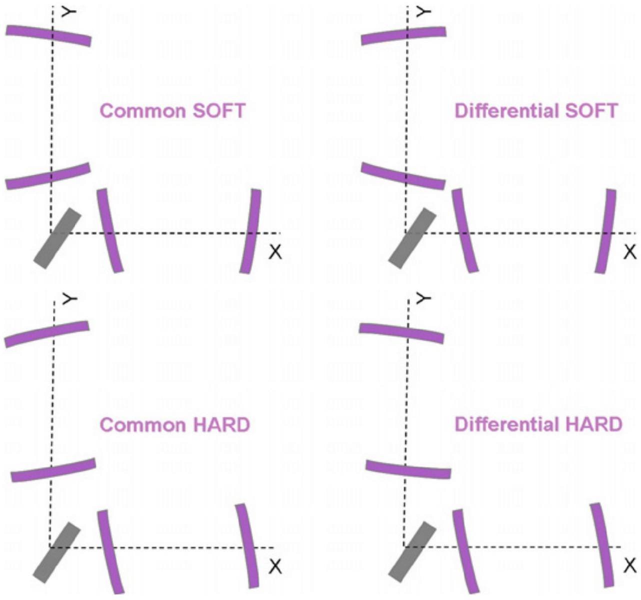
Optimize with noise projections through loops
Iterate design after measuring noise in plant

Why:
Modeling is inexact

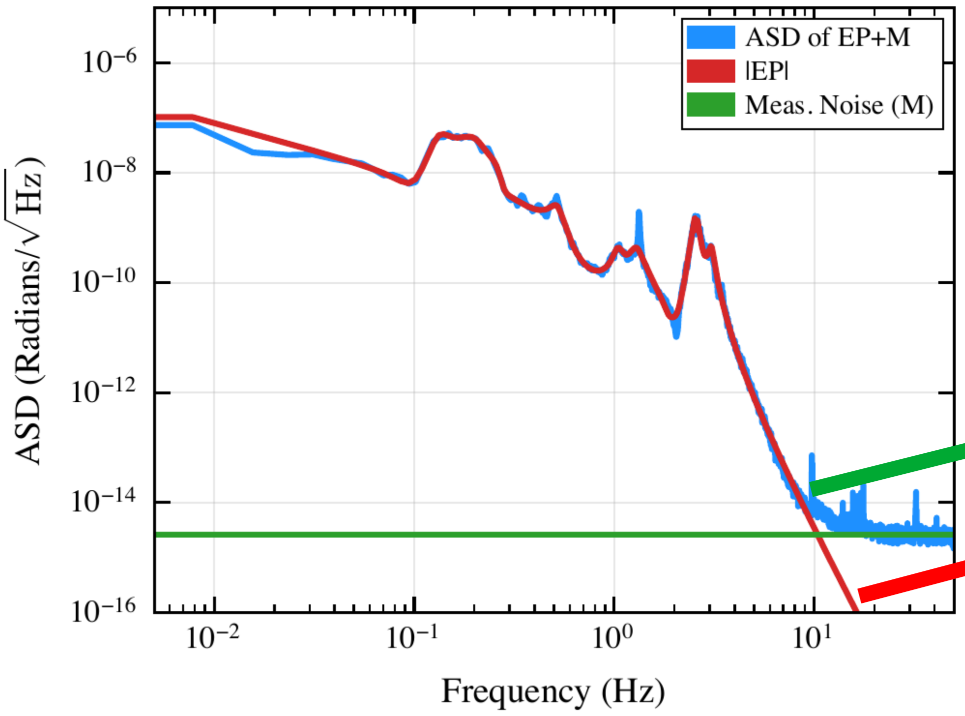
LIGO's noise spans MANY orders of magnitude

Our tooling is efficient for this
Workflow → Rapid operations

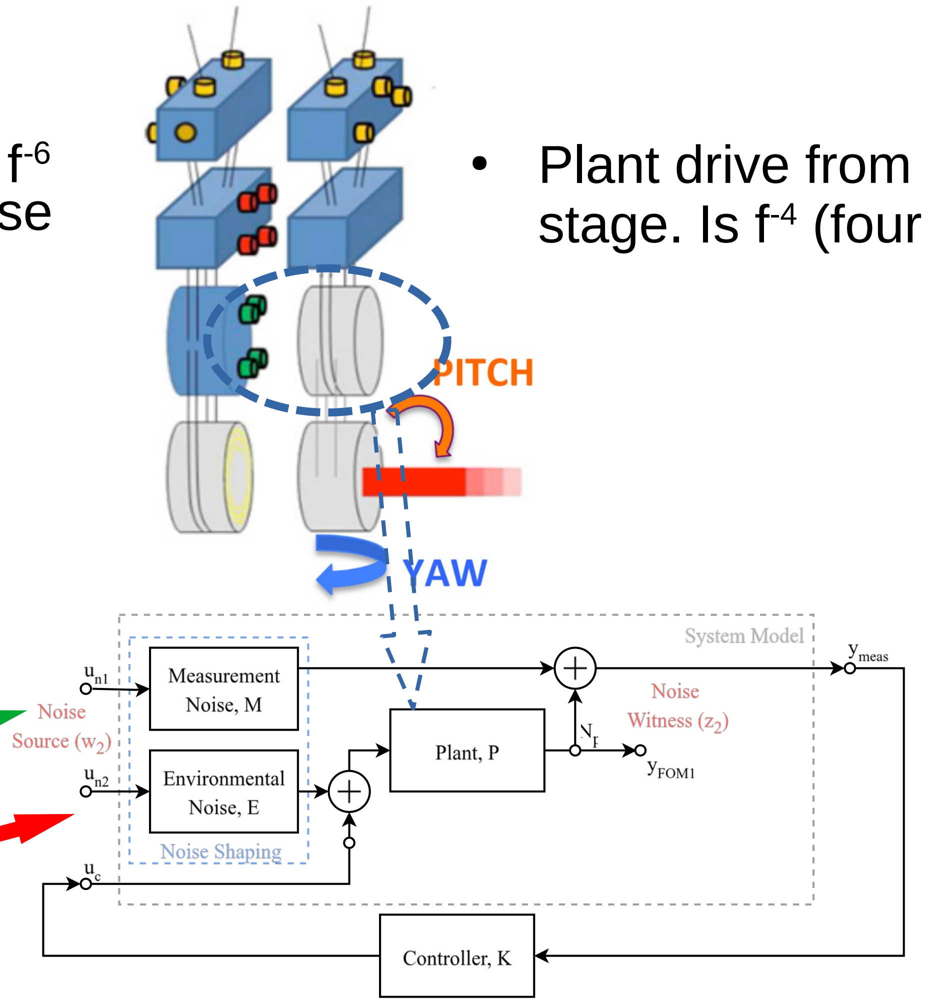




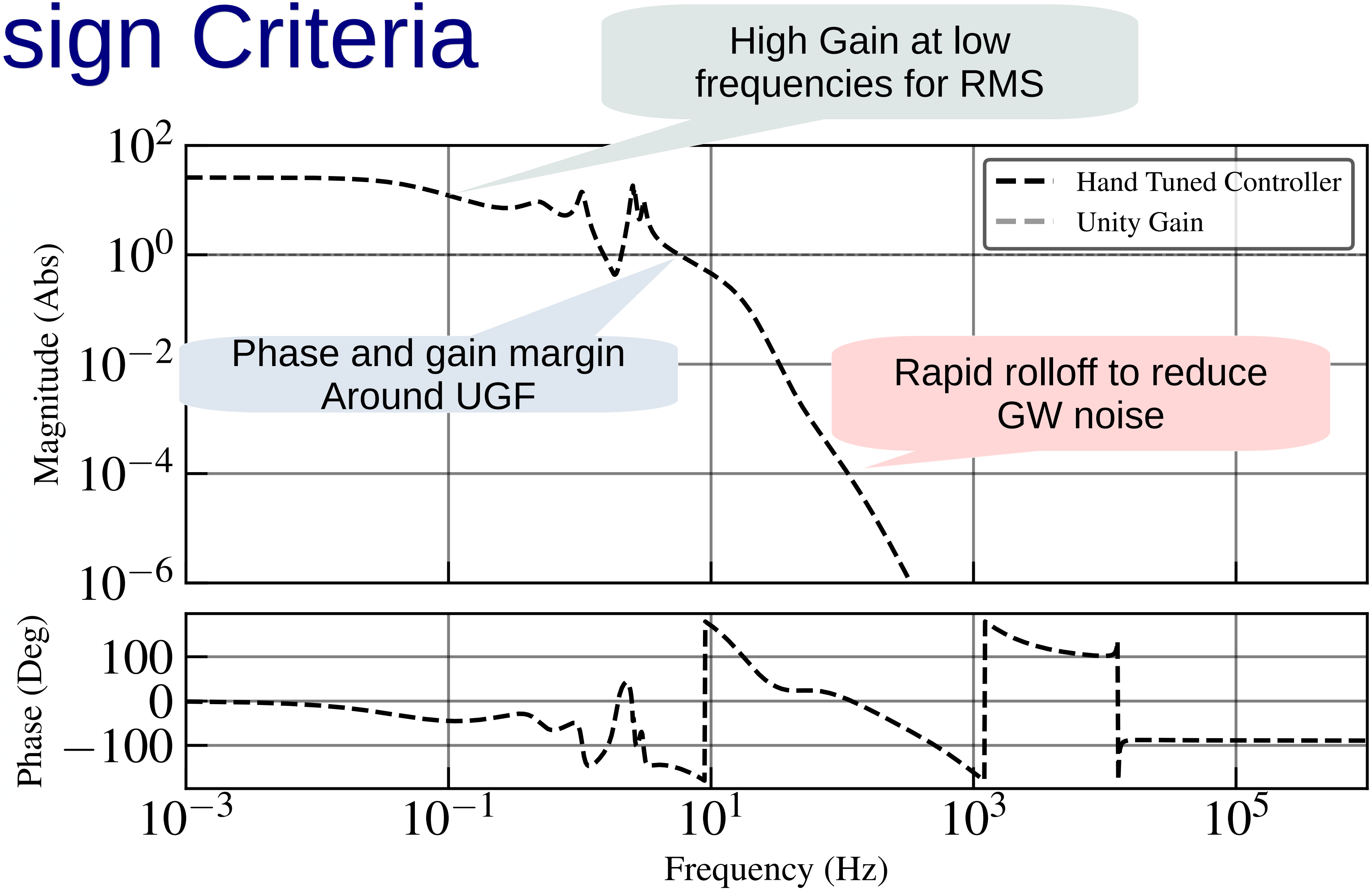
- System noise shaped, sysID fit. Rolloff of f^{-6} with 6 orders of magnitude to sensing noise



- Plant drive from uppe
stage. Is f^{-4} (four pole



Design Criteria



Inertia

- "The weights or objective is arbitrary"

  (no, we have a canonical figure of merit!)

- "It's complicated"

  (not for SISO systems, which require all the same design inputs)

- "$\mathscr{H}_2$ makes marginally stable loops"

  (fixed, add a simple and universal h-inf bound)

- "$\mathscr{H}_\infty$ doesn't optimize for noise"

  (fixed, use it as a bound)

- "AI controllers will render this obsolete"

  (how will you benchmark AI without optimal linear controller ?)

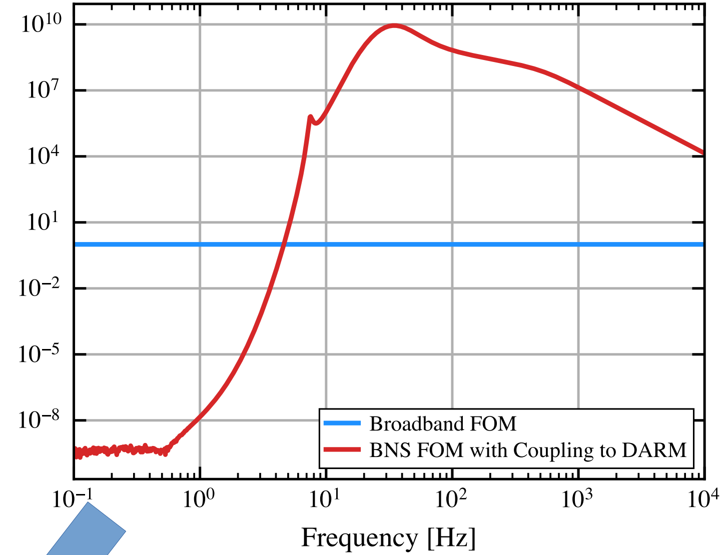
Linearized SNR
Or “observing range”

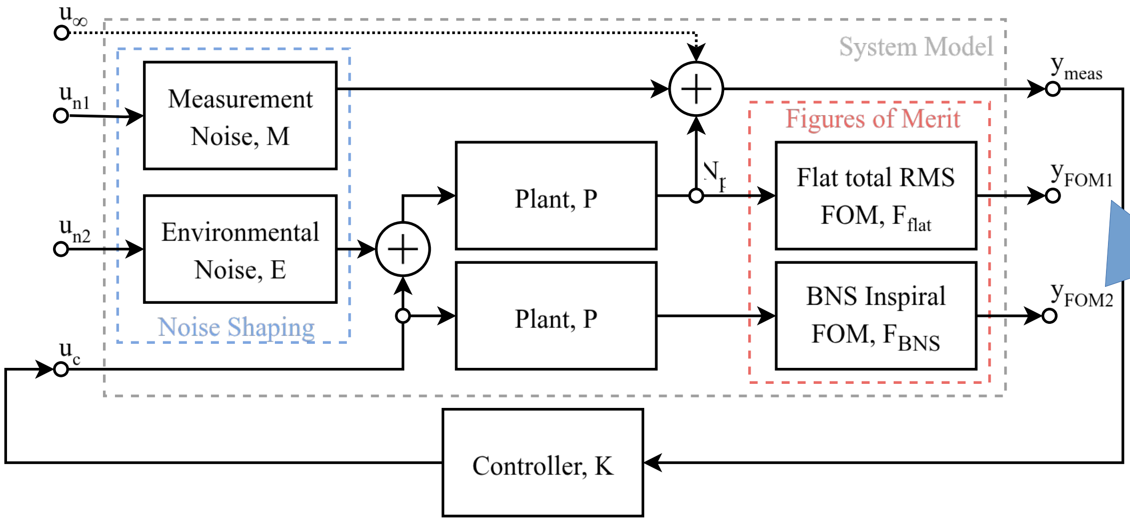
$$\textsc{Snr}^2 = 4\int_0^\infty \frac{h^2(f)}{S_{hh}(f)}\mathrm{d}f - 4\int_0^\infty \frac{h^2(f)}{S_{hh}^2(f)} C_{h\theta}^2 S_{\theta\theta}^2 \mathrm{d}f$$

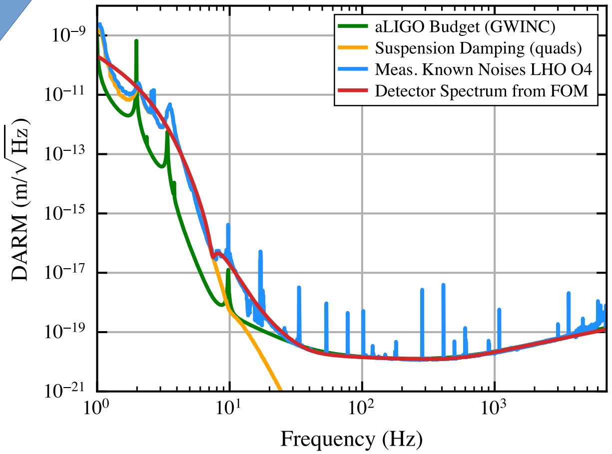
18 orders
Of magnitude

Canonical figure of merit
as L2 weighting filter

$$|F_{\mathrm{BNS}}(f)|^2 \equiv 4\frac{h^2(f)}{S_{hh}^2(f)} C_{h\theta}^2$$







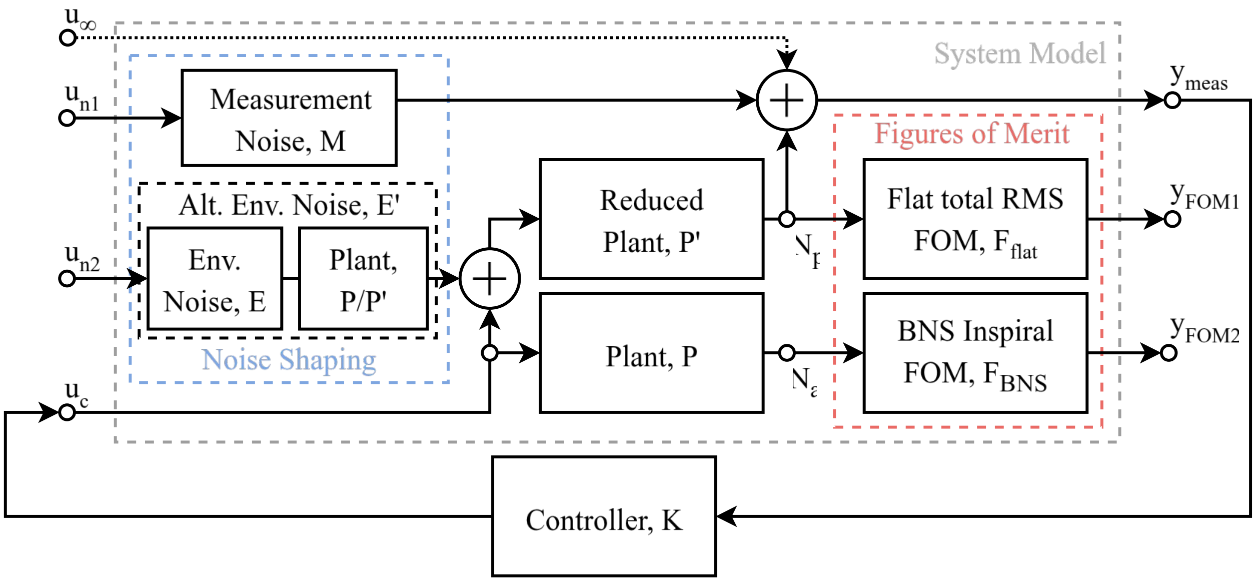
Technical Fixes I

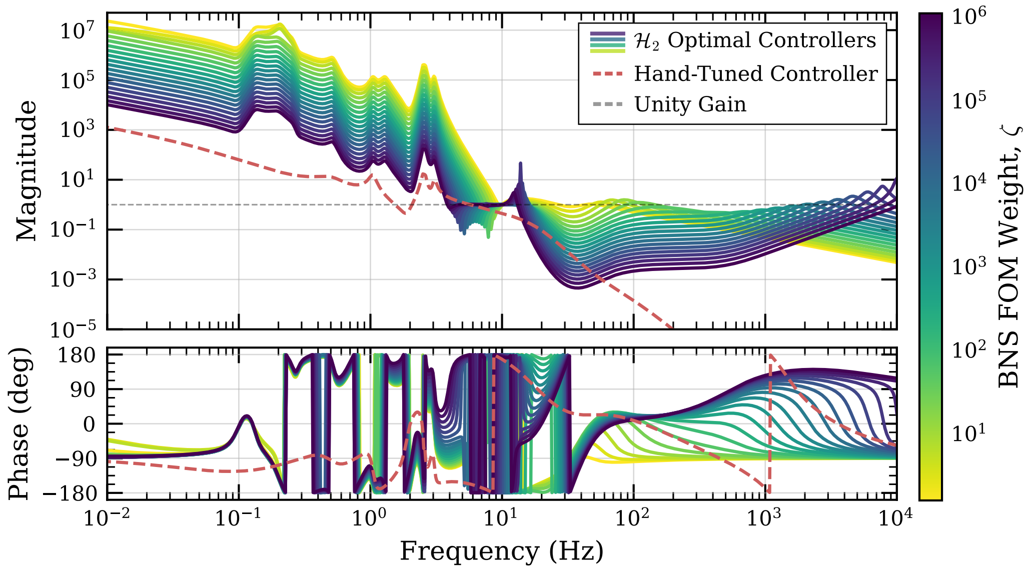
Additionally

- plant must use inner/outer factorization
  - P' has unstable poles and nonmin-phase zeros

- plant must be split for actuator noise.

- Statespace is *extremely* non-normal
  - very careful numerical balancing
  - Grammian order reduction helps a lot (AB09ND slicot)

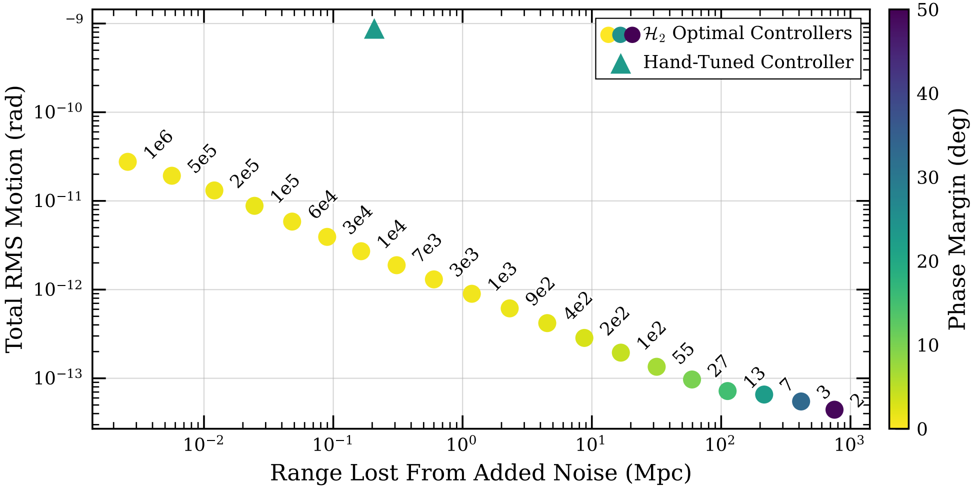


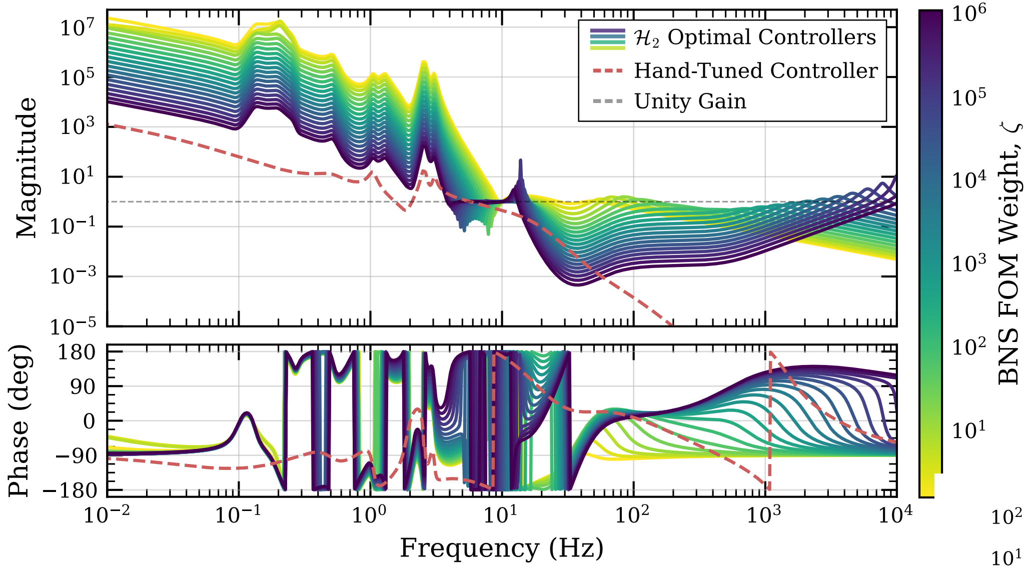


Scroll the relative weight of two FOMs

Solves for the Pareto-front on Bilinear noise

H2 solver unsurprisingly has very Bad phase margins with highly-shaped Input (noise) weights and output (FOM) weights





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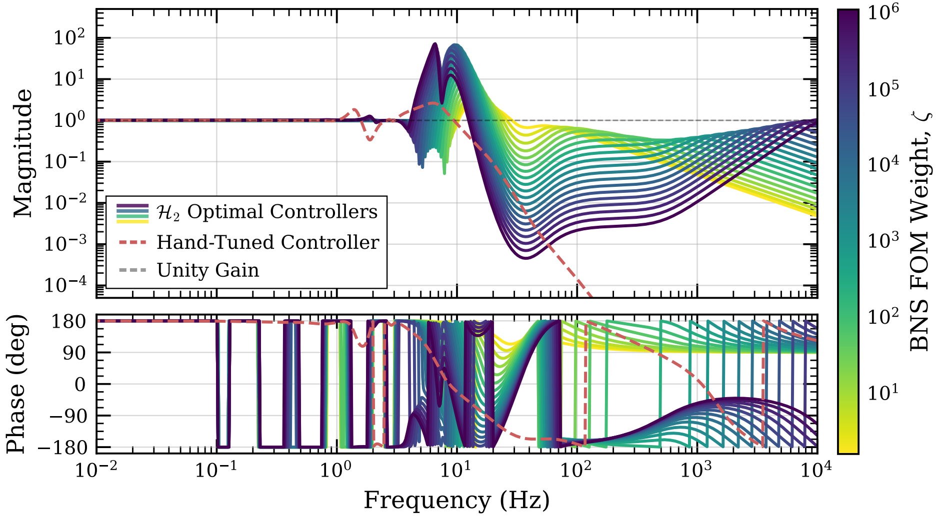


H2 solver unsurprisingly has very Bad phase margins with highly-shaped Input (noise) weights and output (FOM) weights

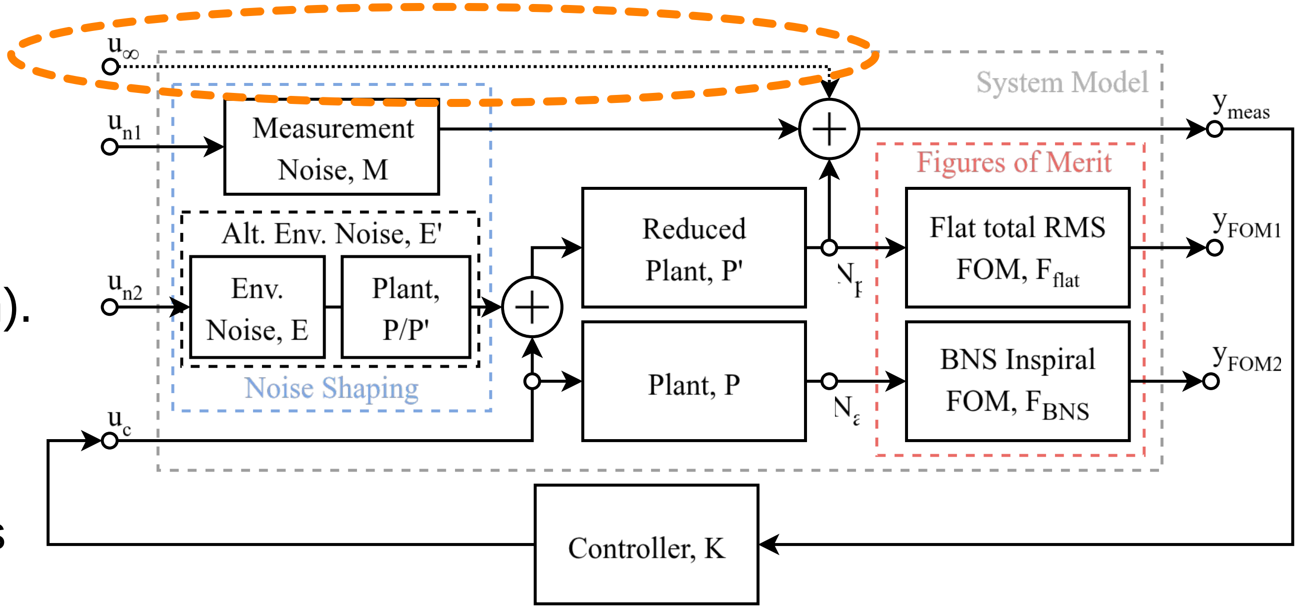
Technical Fixes II

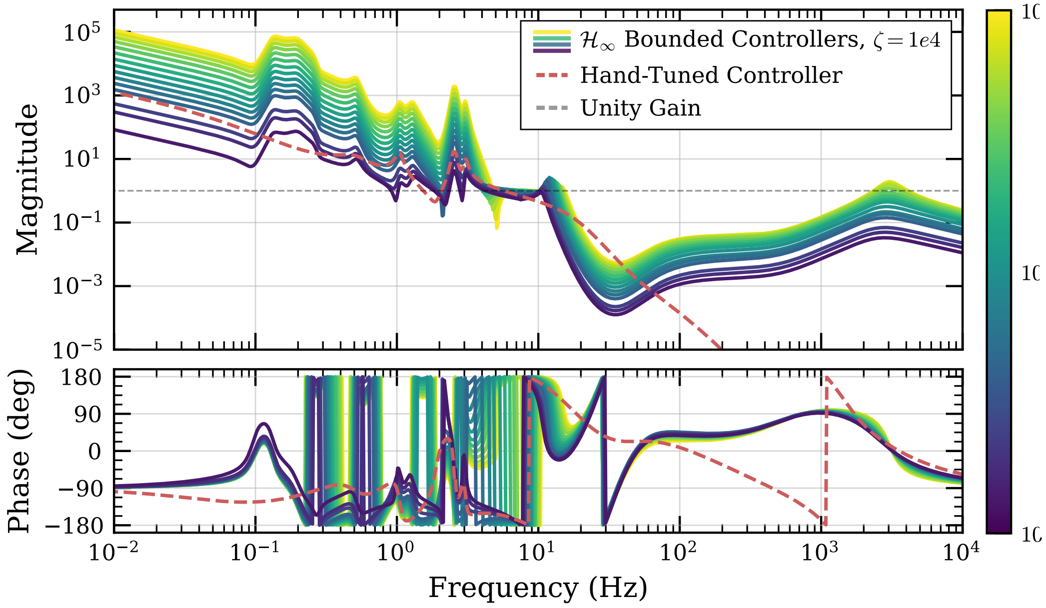
The u-∞ input to FOM can be used as an H-∞ constraint on Gain peaking (phase margin)!

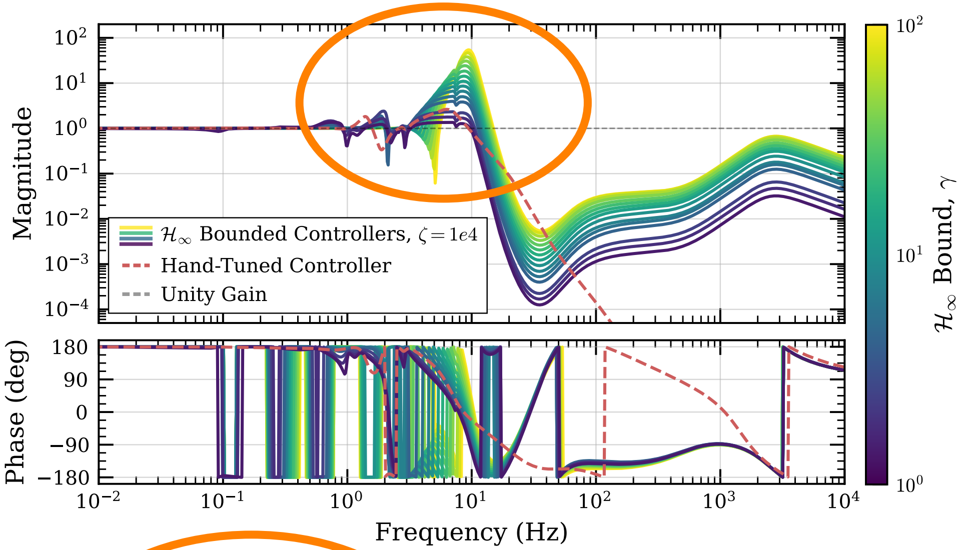
Use formulas of Bernstein-Haddad for 3 coupled algebraic-Riccati equations (z-form). (J. Doyle proved sufficiency)

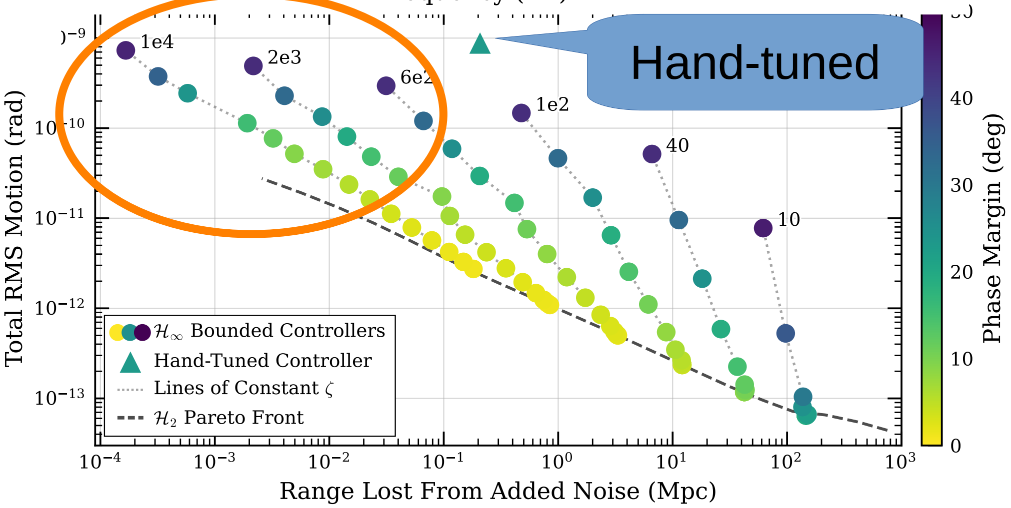
Algebraic (deflating subspace) solver with iterative convergence is reliable.

Can’t (yet) use convex opt. including Matlab’s H2Hinfopt.









Can now generate Pareto-front as a function
Of the **phase margin** as well.

Gain margin is generally monotonic with
Phase margin (not shown)

Results

- Now able to objectively model LIGO’s loop noises through fitted (input) noise weights.

- Useful (canonical) figures of merit also through output weighting functions

- Generated reliable numerical code for state-spaces with large (full) dynamic range and highly non-normal matrices

- Still limited by MIMO nonparametric sysID and modeling issues (plant isn’t as diagonal as assumed)

- with SISO demonstrated, can move to full MIMO solve.

- need to migrate slycot to Julia’s Descriptor Systems library
(more from Andreas Varga)
Sound numerical methods are paramount!